Kepler's Laws and the Moon

Astronomy Laboratory Exercise

5

Learning Objectives

In this laboratory exercise, students will:

- Explore the concept of orbiting bodies.
- Explore Kepler's three laws of planetary motion.

> ***Definitions:*** eccentricity, semi-major axis, semi-minor axis

Introduction

On July 19th, 1967 the *Explorer 35* robotic spacecraft was launched from Cape Kennedy (in Florida) and placed into orbit around the Moon. The purpose of the mission was to gather scientific information about the Moon's magnetic field, the interaction of high-energy particles (given off by the Sun) with the Moon, and details about X-rays given off by the Sun. The 230-kilogram spacecraft (see Figure 1) was placed into a nearly 12 hour elliptical orbit around the Moon. The spacecraft continued to operate successfully until it was turned off in June of 1973.

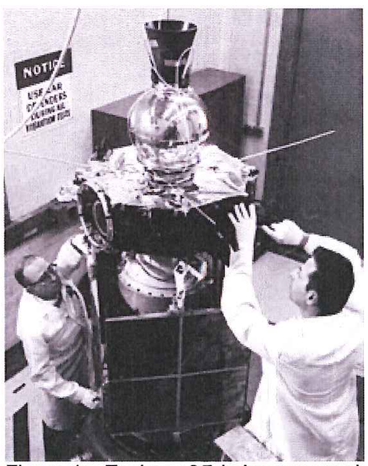
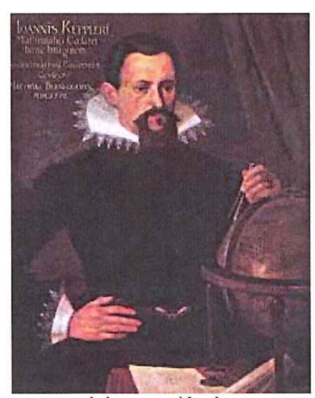


Figure 1: Explorer 35 being prepared for launch. (Courtesy of NASA Goddard Space Flight Center)

Figure 2: The Moon.

The beginning of our understanding of how objects orbit other bodies began in the early part of the 17th Century when Johannes Kepler discovered that there are three basic rules that describe the nature of planetary orbits. Using astronomical data gathered by his predecessor (an astronomer named Tycho Brahe) Kepler was able to determine that the planets in our solar system orbit the Sun in a fashion that is described by three basic rules. These three rules described the shape of the planetary orbits, the speed of the orbiting bodies in their orbits, and the periods and distances of the planets in their orbits around the Sun. Later in the 17th Century, Isaac Newton would discover that it was the force of gravity that was governing Kepler's Laws. Using his mathematical description of gravity and the forces of nature that govern moving bodies, Newton would be able to formulate the physics that would ultimately be used to place the *Explorer 35* spacecraft into lunar orbit, nearly 300 years later.



Johannes Kepler

In this exercise you will plot the orbit of *Explorer 35* and examine the nature of its orbit. From your analysis we will see first hand the three laws of planetary motion that were first described by Kepler nearly 400 years ago.

Procedure

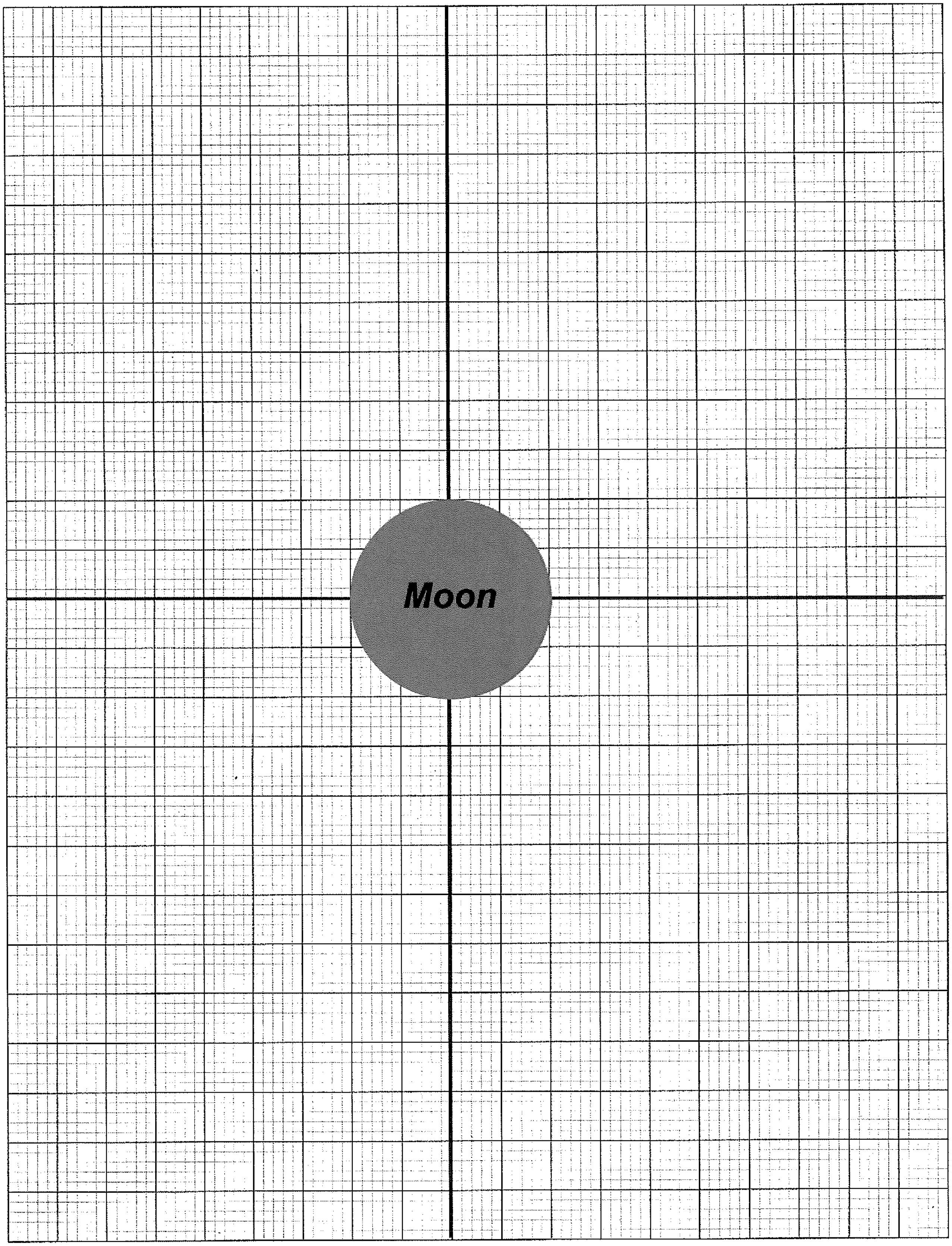
In Table A (below) the orbital information for *Explorer 35* is given. The table contains the x,y coordinates for the location of the spacecraft in its orbit around the Moon. The coordinates are listed for a period of nearly 12 hours and are listed at 15-minute intervals. The unit of measure is in lunar-radii (units equal to the radius of the Moon). The center of the Moon is located at the x,y coordinate of 0,0.

1. Using the sheet of graph paper on the next page, plot the orbit of *Explorer 35*. ***Graph the data points as accurately as possible. Your accuracy is significant to the success of this exercise.*** (*Note: You do not need to label the time next to each data point).*

Table A

Elapsed Time	x (lunar radii)	y (lunar radii)	Elapsed Time	x (lunar radii)	y (lunar radii)
$0^h 00^m$	-3.62	1.04	$5^h 45^m$	0.00	4.74
$0^h 15^m$	-3.46	0.63	$6^h 00^m$	-0.27	4.86
$0^h 30^m$	-3.25	0.20	$6^h 15^m$	-0.56	4.95
$0^h 45^m$	-2.97	-0.22	$6^h 30^m$	-0.84	5.01
$1^h 00^m$	-2.60	-0.65	$6^h 45^m$	-1.12	5.03
$1^h 15^m$	-2.14	-1.03	$7^h 00^m$	-1.38	5.04
$1^h 30^m$	-1.55	-1.37	$7^h 15^m$	-1.64	5.00
$1^h 45^m$	-0.85	-1.58	$7^h 30^m$	-1.89	4.95
$2^h 00^m$	-0.03	-1.59	$7^h 45^m$	-2.14	4.87
$2^h 15^m$	+0.78	-1.32	$8^h 00^m$	-2.37	4.77
$2^h 30^m$	1.45	-0.79	$8^h 15^m$	-2.59	4.65
$2^h 45^m$	1.87	-0.11	$8^h 30^m$	-2.80	4.50
$3^h 00^m$	2.09	+0.58	$8^h 45^m$	-2.99	4.33
$3^h 15^m$	2.16	1.22	$9^h 00^m$	-3.17	4.14
$3^h 30^m$	2.11	1.82	$9^h 15^m$	-3.33	3.93
$3^h 45^m$	1.99	2.35	$9^h 30^m$	-3.49	3.69
$4^h 00^m$	1.82	2.81	$9^h 45^m$	-3.59	3.42
$4^h 15^m$	1.61	3.22	$10^h 00^m$	-3.69	3.15
$4^h 30^m$	1.37	3.59	$10^h 15^m$	-3.77	2.85
$4^h 45^m$	1.11	3.90	$10^h 30^m$	-3.81	2.52
$5^h 00^m$	0.85	4.16	$10^h 45^m$	-3.83	2.20
$5^h 15^m$	0.58	4.40	$11^h 00^m$	-3.81	1.83
$5^h 30^m$	0.28	4.58	$11^h 15^m$	-3.76	1.46

Courtesy NASA Goddard Space Flight Center


Moon

Ellipse Geometry

An ellipse is a unique shape in geometry. It is defined as a set of points that are at a fixed distance from two primary points (each primary point is called a "*focus*"). The sum of the two lines joining any point on the ellipse and the two *foci* (plural for focus) is always the same (see Figure 3 to the right). For the orbit of *Explorer 35*, the Moon is located at one focus.

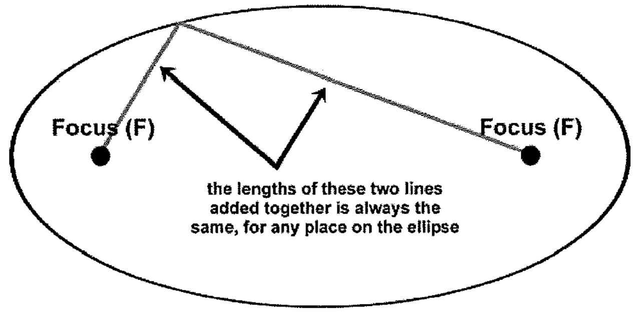


Figure 3: definition of the ellipse.

The longest axis running across the ellipse is called the major axis. This axis always runs through the two foci (see Figure 4 to the right).

2. Determine where the major axis is on your graph of *Explorer 35's* orbit about the Moon. Draw a line on your graph that represents the major axis. Label the axis *"major axis".*

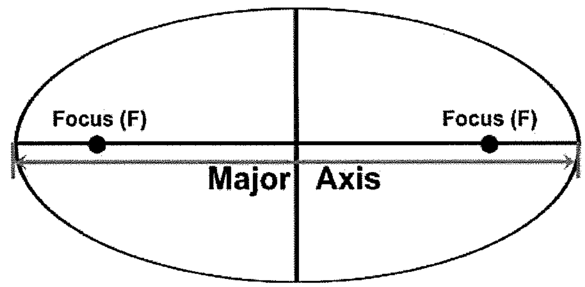


Figure 4: Major axis of the ellipse.

The shortest axis running across the ellipse is called the minor-axis (see Figure 5 to the right).

3. Determine where the minor axis is on your graph of *Explorer 35's* orbit about the Moon. Draw a line on your graph that represents the minor axis. Label the axis *"minor axis".*

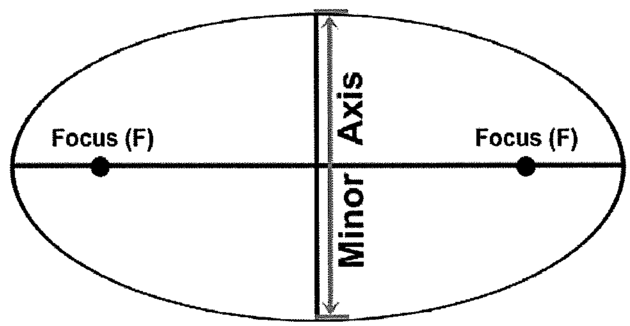


Figure 5: Minor axis of the ellipse.

In the ellipse, half of the major axis is called the ***semi-major axis***. Similarly, half of the minor axis is called the ***semi-minor axis*** of the ellipse (see Figure 6 to the right).

4. Determine the semi-major axis and the semi-minor axis of the ellipse that represents *Explorer 35's* orbit about the Moon, and label it on your graph.

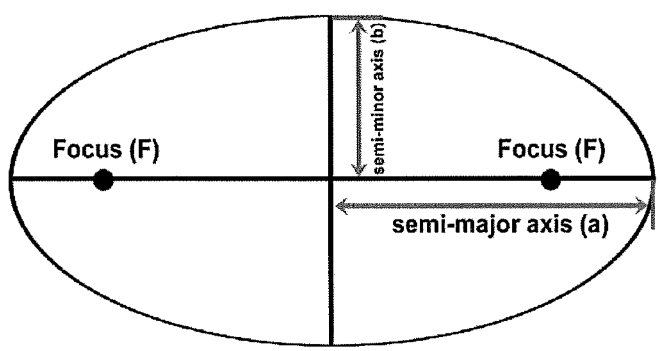


Figure 6: The semi-major and semi-minor axes of the ellipse.

The overall shape of the ellipse is determined by a term that is called the ***eccentricity*** of the ellipse. It is represented by dividing the length of the semi-major axis by the distance between one focus and the center of the ellipse (see Figure 7 to the right) and is calculated with the following equation:

$$eccentricity = \frac{c}{a}$$

Equation 1

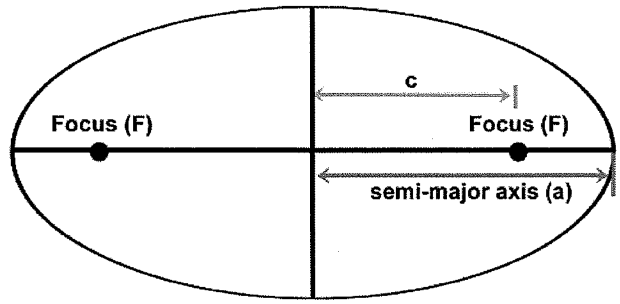


Figure 7

5. Measure the length of the semi-major axis in millimeters (this is <u>a</u> in Equation 1). *Record your answer in Data Table 1 on the Data Sheet.*

6. Measure the distance from the center of the Moon (remember that this is one focus) to where the major axis and minor axis cross (this is <u>c</u> in Equation 1). *Record your answer in Data Table 1 on the Data Sheet.*

7. Divide the two to get the eccentricity of the orbit of *Explorer 35*. *Record your answers in Data Table 1 on the Data Sheet.*

Question 1a: What happens to the shape of the ellipse if the distance between the two foci is larger but the major axis does not change its size? *Write your answer in Data Table 1 on the Data Sheet.*

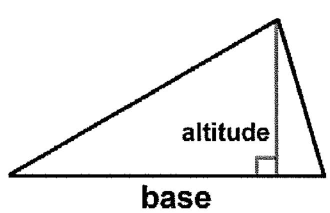
Question 1b: What happens to the value of the eccentricity in this case? *Write your answer in Data Table 1 on the Data Sheet.*

Question 2a: What shape do you think the ellipse would have if the distance between the two foci was zero? *Write your answer in Data Table 1 on the Data Sheet.*

Question 2b: What value would the eccentricity have in this case? *Write your answer in Data Table 1 on the Data Sheet.*

Kepler's *<u>First</u>* Law says that all orbiting bodies have orbits in the shape of an ellipse with the body being orbited located at one focus.

8. Pick four consecutive data points along the elliptical orbit for *Explorer 35*. Draw a triangle from the center of the Moon, to the first point you chose, to the last of the four points you chose and then back to the center of the Moon (see Figure 8). Determine the area of the triangle using the figure and Equation 2 shown below: *Record your answer in Data Table 2 on the Data Sheet.*



$$area\ of\ triangle = \tfrac{1}{2}(base)(altitude)$$

Equation 2

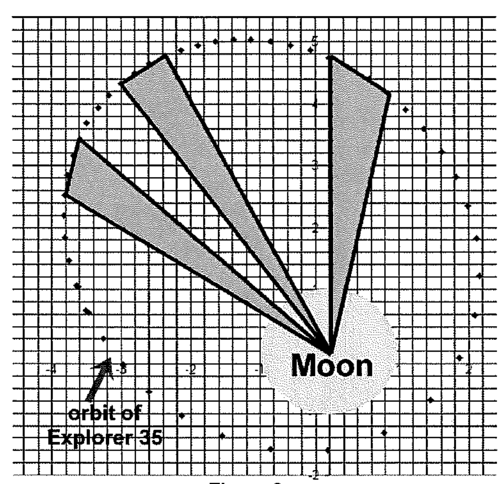


Figure 8

9. Create <u>two more</u> triangles and calculate the areas of each triangle. *Record your answers in Data Table 2 on the Data Sheet.*

Question 3a: If you chose four consecutive data points for each triangle, then how much time (how many minutes) did the spacecraft travel to cover each area? *Record your answer in Data Table 2 on the Data Sheet.*

Question 3b: Are all three triangles the same time interval? *Record your answer in Data Table 2 on the Data Sheet.*

Question 4: What do you notice about the areas (factor in a reasonable amount of plotting/measurement error) of the three triangles that you chose? *Record your answer in Data Table 2 on the Data Sheet.*

Question 5: Is there a relationship between the *intervals of time* and the *areas of the triangles* for the three triangles you chose? *Record your answer in Data Table 2 on the Data Sheet.*

Question 6: If all three portions of the orbit you chose cover the same amount of time, yet the spacecraft makes a different amount of progress along the orbital path, what does this mean about the *speed* of the spacecraft as it makes it's orbit around the Moon? *Record your answer in Data Table 2 on the Data Sheet.*

Kepler's *Second* Law describes equal areas covered by an orbiting body in equal intervals of time.

Question 7a: Looking at the orbital path of *Explorer 35* and your answers to the questions above, where would you say that the spacecraft moves *fastest* in its orbit? *Write your answer in Data Table 2 on the Data Sheet.*

Question 7b: Where does the spacecraft move *slowest* in its orbit? *Write your answer in Data Table 2 on the Data Sheet.*

10. Using the graph of *Explorer 35's* orbit or the data set, determine how long (in minutes) it takes the spacecraft to orbit the Moon. This is called the orbital period of the spacecraft. *Record your answer in Data Table 3 on the Data Sheet.*
11. Convert the orbital period of *Explorer 35's* (calculated in step 10) to years. Use Appendix A if needed. *Record your answer in Data Table 3 on the Data Sheet.*
12. The units on your graph are in lunar radii. The gray circle on your graph represents the diameter of the moon. Using your ruler measure how many millimeters there are in one lunar diameter.
13. The diameter of the Moon is 3,474 kilometers. Using your answer from question #12, determine how many kilometers/millimeter are on your graph. *Record your answer in Data Table 3 on the Data Sheet.*
14. Using the scale you determined in step 13, convert your measurement for the semi-major axis of *Explorer 35's* orbit from step 5 from millimeters to kilometers. *Record your answer in Data Table 3 on the Data Sheet.*
15. Convert the size of the semi-major axis for the orbit of *Explorer 35* from kilometers to Astronomical Units. *Record your answer in Data Table 3 on the Data Sheet.*
16. Square the orbital period of *Explorer 35* (in years; determined in step 11). *Record your answer in Data Table 3 on the Data Sheet.*
17. Cube the size of the semi-major axis of *Explorer 35's* orbit (in AUs; determined in step 15). *Record your answer in Data Table 3 on the Data Sheet.*

Isaac Newton showed that Kepler's laws are a consequence of gravity. In this application, the orbit of *Explorer 35* is governed by the gravity of the Moon. The force of gravity is determined by the masses of the objects that are gravitationally attracting each other (this concept will be explored further in a future exercise).

18. Taking the mass of the Moon into consideration multiply the answer to step 17 by 2.82×10^7. *Record your answer in Data Table 3 on the Data Sheet.*

Question 8: What do you notice when you compare the square of the orbital period of *Explorer 35* (determined in step 16) with number you calculated in step 18? *Write your answer in Data Table 3 on the Data Sheet.*

Kepler's *Third* Law says that the square of the orbital period of an orbiting body is equal to the cube of the orbiting body's semi-major axis.

It was later shown by Isaac Newton that Kepler's Laws are applicable to *any* two orbiting bodies.

Kepler's Laws and the Moon

Name:

Astronomy Laboratory Data Sheet

Data Table 1

a (on your graph)	mm
c (on your graph)	mm
eccentricity	
Question 1a	
Question 1b	
Question 2a	
Question 2b	

Data Table 2

Area of triangle 1	mm^2
Area of triangle 2	mm^2
Area of triangle 3	mm^2
Question 3a	
Question 3b	
Question 4	
Question 5	
Question 6	
Question 7a	
Question 7b	

Data Table 3

Period of Explorer 35	minutes
Period of Explorer 35	years
Semi-major axis	mm
Scale of your graph	km/mm
Semi-major axis	km
Semi-major axis	AUs
$(\text{period})^2$	
$(\text{semi-major axis})^3$	
Step 18	
Question 8	