The Mass of Jupiter

Astronomy Laboratory Exercise

6

Learning Objectives

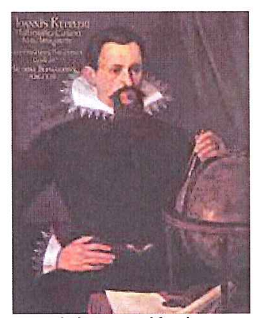
In this laboratory exercise, students will:

- Explore the concept of orbiting bodies
- Learn how to determine the mass of a celestial object
- Learn how the Earth, Jupiter and the Sun compare in mass, size and volume.

> ***Definitions:*** mass, weight, volume
>
> ***Review concepts of*:** Kepler's 3rd Law

Introduction

One of the most fundamental properties of a celestial object is its ***mass***. The *mass* of an object is defined as *how much material the object is composed of.* Unfortunately, those who are new to science will frequently confuse the concept of mass and ***weight***. The *weight* of an object is actually a *force*. It is related to the gravitational force of attraction between two bodies. When you step on a scale to measure your weight, the indicator on the scale is reporting to you a number that represents how much the Earth is gravitationally tugging on your body. According to Isaac Newton's Universal Law of Gravitation, the strength of the force of gravity acting between two objects depends on the size of the masses of the two objects. Therefore, objects on Earth that are more massive, will also weigh more. This allows us to determine the mass of an object, in essence, by weighing it. However, in space, there is no convenient means in which to weigh a celestial object. Another method must be used.


Johannes Kepler

In the early part of the 17th Century, Johannes Kepler discovered that there is a relationship between the orbital period of a planet and the orbital distance of that planet (see the *"Kepler's Laws and the Moon"* exercise). This relationship, known as Kepler's 3rd Law, states that the square of the period of a planet in its orbit about the Sun is equal to the cube of its orbital distance from the Sun (see Equation 1). *It was later determined that this relationship is valid for* *<u>any</u>* *two orbiting bodies.*

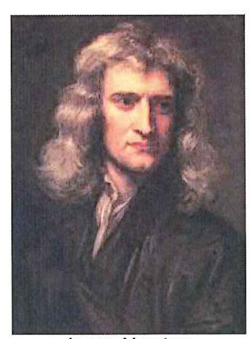
$$p^2 = d^3$$

Equation 1

Later in the 17th Century, Isaac Newton determined that Kepler's 3rd Law is actually the result of the gravitational interaction of the two orbiting bodies. Using his law of gravitation, Newton was able to show that it was the force of gravity that linked the orbital period and the orbital distance together (see Equation 2). In his equation, Newton relates the masses of the orbiting bodies to the other orbital information (orbital period and orbital distance).

$$p^2 = \frac{4\pi^2}{G(m_1 + m_2)} d^3$$

Equation 2


Isaac Newton

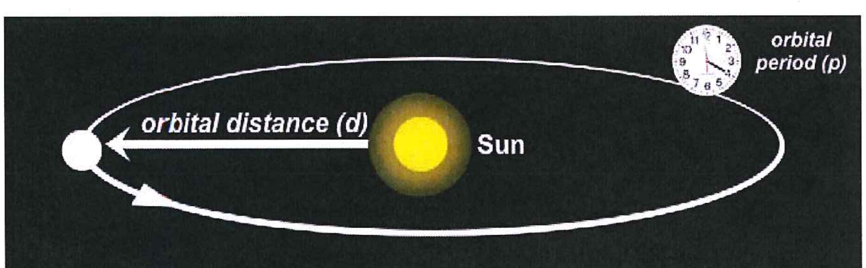


Figure 1: Kepler's 3rd Law

This relationship (Equation 2) is called *Newton's version of Kepler's 3rd Law* and is the tool that we can use to measure the masses of celestial objects.

To demonstrate how this tool (equation) works, you will determine the mass of the planet ***Jupiter***. For *Newton's version of Kepler's 3rd Law* to work, the object whose mass you are measuring must have another body orbiting it. In essence it is this gravitational dance of an orbit that is what you are measuring and through physical laws, you can determine the masses involved.

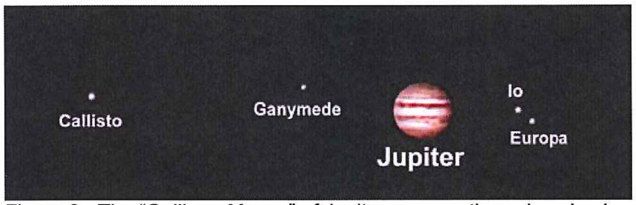


Figure 2: The "Galilean Moons" of Jupiter as seen through a simple telescope.

In 1609 it was Galileo who discovered that Jupiter was being orbited by four large moons. In his honor, these moons are referred to as the *Galilean moons of Jupiter*. Observers on Earth can easily see these four large moons as they move in their orbits around Jupiter (see Figure 2). Each of Jupiter's moons is a unique world unto itself (see Figure 3) and several spacecraft have been sent to Jupiter over the years to study these moons in detail.

Over a period of time, these moons can be seen to move from one side of Jupiter to the other as they complete their orbits. Therefore, the orbital *period* of each moon can be easily determined. By watching the moons in their orbits, observers can also determine the *distance* each moon is in its orbit around Jupiter. With this information, *Newton's version of Kepler's 3rd Law* can be used to determine the mass of Jupiter itself.

Figure 3: The "Galilean Moons" of Jupiter. NASA/JPL

One difficulty with using *Newton's version of Kepler's 3rd Law* (Equation 2) is that this formula actually calculates the masses of <u>both</u> Jupiter *AND* the moon that is being observed (i.e. the *SUM* of the two masses, see Equation 3). However, there is a simplification that can be made.

$$(M_{Jupiter} + m_{moon}) = \frac{4\pi^2}{G}\frac{d^3}{p^2}$$

Equation 3

Where G is the Universal Gravitational Constant, d is the orbital distance of the moon from Jupiter, and p is the orbital period of the moon.

As it turns out, Jupiter is a planet that is many times more massive than any one of its moons. Therefore, it is possible to *ignore* the mass of the moon (since it is so *insignificantly small* in comparison to Jupiter's mass) and drop it out of the left side of Equation 3. The final result is an equation that very closely <u>approximates</u> the mass of Jupiter while still using the orbital information from the orbiting moon.

$$M_{Jupiter} \approx \frac{4\pi^2}{G}\frac{d^3}{p^2}$$

Equation 4

The only task that is left is to observe the orbital period and orbital distance of any one of Jupiter's moons and to use this modified form of *Newton's version of Kepler's 3rd Law* to determine the mass of the planet Jupiter.

Procedure

Table A and *Table B* (on the next page) contain the orbital data for the Galilean moons *Ganymede* and *Europa* (see Figure 3). The tables show (1) the date of the observation, (2) the day number, (3) the time of the observation (in Universal Time) and (4) the position of the moon in its orbit around Jupiter. The negative and positive signs for the positions indicate that the moon is either on the east side (negative numbers) or the west side (positive numbers) of Jupiter as seen by observers on Earth. The unit for the positions of the moons is measured in *Jupiter diameters* (i.e. a measurement of –3.0 would mean that the moon was on the east side of Jupiter at a distance that is equal to three times the diameter of Jupiter in distance).

1. Create two separate graphs and plot the data from both Table A and Table B. In your graphs, use the <u>*day number*</u> as the x-axis and the <u>*position*</u> (in Jupiter diameters) as the y-axis.

Table A

Ganymede			
DATE	DAY	TIME	POSITION (Jupiter diameters)
10/29	1.0	0:00	-3.04
10/29	1.5	12:00	+0.15
10/30	2.0	0:00	+3.30
10/30	2.5	12:00	+5.83
10/31	3.0	0:00	+7.28
10/31	3.5	12:00	+7.33
11/1	4.0	0:00	+6.03
11/1	4.5	12:00	+3.59
11/2	5.0	0:00	+0.46
11/2	5.5	12:00	-2.75
11/3	6.0	0:00	-5.45
11/3	6.5	12:00	-7.13
11/4	7.0	0:00	-7.30
11/4	7.5	12:00	-6.38
11/5	8.0	0:00	-4.10
11/5	8.5	12:00	-1.05
11/6	9.0	0:00	+2.20
11/6	9.5	12:00	+5.02
11/7	10.0	0:00	+6.90
11/7	10.5	12:00	+7.48
11/8	11.0	0:00	+6.65
11/8	11.5	12:00	+4.58

Table B

Europa			
DATE	DAY	TIME	POSITION (Jupiter diameters)
10/29	1.00	0:00	-3.35
10/29	1.25	6:00	-1.60
10/29	1.50	12:00	+0.48
10/29	1.75	18:00	+2.43
10/30	2.00	0:00	+3.91
10/30	2.25	6:00	+4.63
10/30	2.50	12:00	+4.47
10/30	2.75	18:00	+3.45
10/31	3.00	0:00	+1.79
10/31	3.25	6:00	0.00
10/31	3.50	12:00	-2.19
10/31	3.75	18:00	-3.75
11/1	4.00	0:00	-4.62
11/1	4.25	6:00	-4.58
11/1	4.50	12:00	-3.67
11/1	4.75	18:00	-2.05
11/2	5.00	0:00	-0.03
11/2	5.25	6:00	+2.00
11/2	5.50	12:00	+3.64

2. On each graph, the zero line for the y-axis represents the location of the center of Jupiter. Draw a ***smooth curving line*** that passes through each of the data points on your graph. DO NOT CONNECT THE DOTS!

3. Determine the period of the orbit for each moon. Each moon will complete one orbit around Jupiter when the position of the moon has returned to its original starting point in its orbit. To measure this in your graph, measure the time it takes for the moon to go from one place along the curve to the identical position once again. Be very careful and accurate in your measurement. *Record the periods for each moon in Data Table 1 on the Data Sheet.*

4. Determine the orbital distance for each moon. The orbit of Jupiter's moons are edge-on to our perspective from Earth. Each moon will show you its orbital distance when it is at its furthest point (either east or west) on either side of Jupiter as seen by Earthly observers (refer to Figure 1 for an example). Looking at the smooth curving line on your graphs, determine the maximum distance in JDs (it can be either positive or negative) that the moon is from the zero line on your graph (recall that the center of Jupiter is located at the value of zero on the y-axis). (**NOTE:** you do not need to find a place where there is a data point to make your measurement... use the smooth curving line). Be very careful and accurate in your measurement. *Record the orbital distance for each moon in Data Table 1 on the Data Sheet.*

5. For *Newton's version of Kepler's 3rd Law* to work, the orbital period of each moon must be in units of <u>seconds</u>. Convert the period of each moon (measured in step 3) to seconds (consult the conversion table in Appendix A). Do this for both moons. *Record the periods, converted to seconds, in Data Table 1 on the Data Sheet.*

6. Again, for *Newton's version of Kepler's 3rd Law* to work, the orbital distances must be in units of <u>meters</u>. The diameter of Jupiter is 142,990 km. Convert the orbital distance of each moon (measured in step 4) to meters. Do this for both moons. *Record the orbital distances, converted to meters, in Data Table 1 on the Data Sheet.*

7. Using Equation 4 (shown again below), determine the mass of Jupiter. Don't worry about the units if they confuse you, your answer will be in units of *kilograms*.

   Do this step twice (once for each moon). *Record your answers in Data Table 1 on the Data Sheet.*

$$M_{Jupiter} \approx \frac{4\pi^2}{G}\frac{d^3}{p^2}$$

Equation 4

where *d* must be in meters, *p* must be in seconds and **G = 6.673x10^{-11} N-m^2/kg^2**

8. Average the two values of the mass of Jupiter and *record your answer in Data Table 1 on the Data Sheet.*

Now that you see how this concept can work to determine the mass of an object being orbited by another body, lets explore a few other applications of *Newton's version of Kepler's 3rd Law.*

9. Recall that this concept can be used for *ANY* two orbiting bodies. Suppose that we use the Space Shuttle in its orbit about the Earth to measure the mass of the Earth (see Figure 4). The Shuttle orbits the Earth at a distance of 6647 kilometers from the center of the Earth, and takes 89 minutes to complete one orbit about the Earth. Determine the mass of the Earth using this information and Equation 4 (NOTE: you will need to convert the orbital period of the Shuttle into seconds, and the orbital distance into meters). *Record your answer in Data Table 2 on the Data Sheet.*

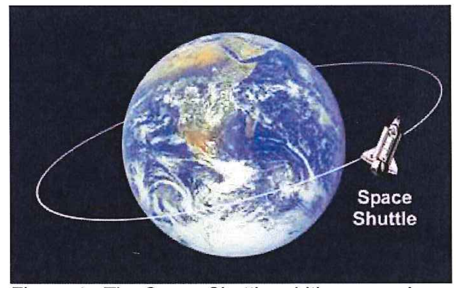


Figure 4: The Space Shuttle orbiting around the Earth.

10. If the Earth takes 365 days to orbit the Sun at a distance of 1 Astronomical Unit, use Equation 4 to determine the mass of the Sun. (NOTE: you will need to convert the orbital period of the Earth into seconds, and the orbital distance of the Earth into meters). *Record your answer in Data Table 2 on the Data Sheet.*

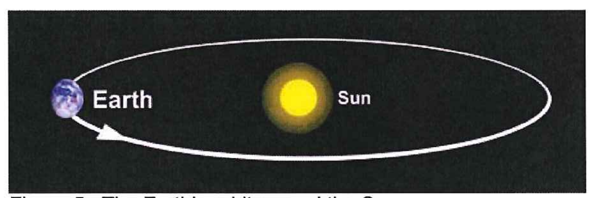


Figure 5: The Earth's orbit around the Sun.

11. A comparison can be made using the masses of the Earth, Jupiter and the Sun. First divide the Mass of Jupiter by the Mass of the Earth (use the masses of Jupiter and the Earth you calculated in this exercise). Next divide the mass of the Sun by the mass of Jupiter. Finally, divide the mass of the Sun by the mass of the Earth. *Record all answers in Data Table 2 on the Data Sheet.*

12. The ***volume*** of a sphere is determined by using Equation 5 (shown to the right). If Jupiter has a radius of 7.15x10^7 meters, determine its volume. *Record your answer on the Data Sheet.* If the Earth has a radius of 6.378x10^6 meters, determine its volume. *Record your answer on the Data Sheet.* If the Sun has a radius of 6.96x10^8 meters, determine its volume. *Record your answer in Data Table 2 on the Data Sheet.*

$$Volume = \frac{4}{3}\pi(radius)^3$$

Equation 5

13. Divide the volume of the Sun by the volume of Jupiter. This is how many "Jupiters" would fit inside of the Sun. Now divide the volume of Jupiter by the volume of the Earth. This is how many "Earths" would fit inside of Jupiter. Finally, divide the volume of the Sun by the volume of Earth. This is how many "Earths" would fill the Sun. *Record all answers in Data Table 2 on the Data Sheet.*

It should now be clear just how massive and large the planet Jupiter is. However, it is simply amazing how massive and large the Sun (a typical star) is compared to ALL planets in our Solar System!

The Mass of Jupiter

Name:

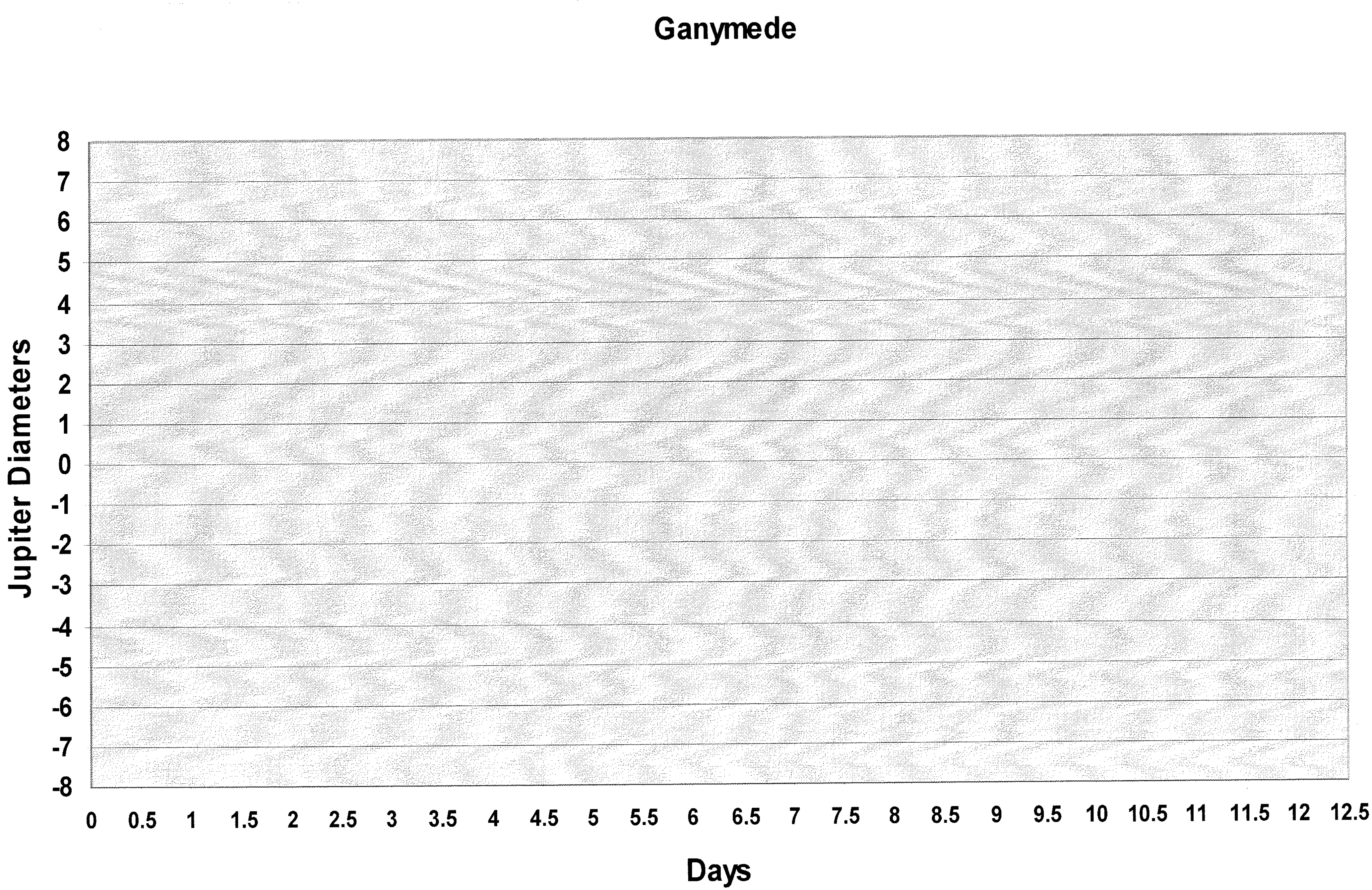
Astronomy Laboratory Data Sheet

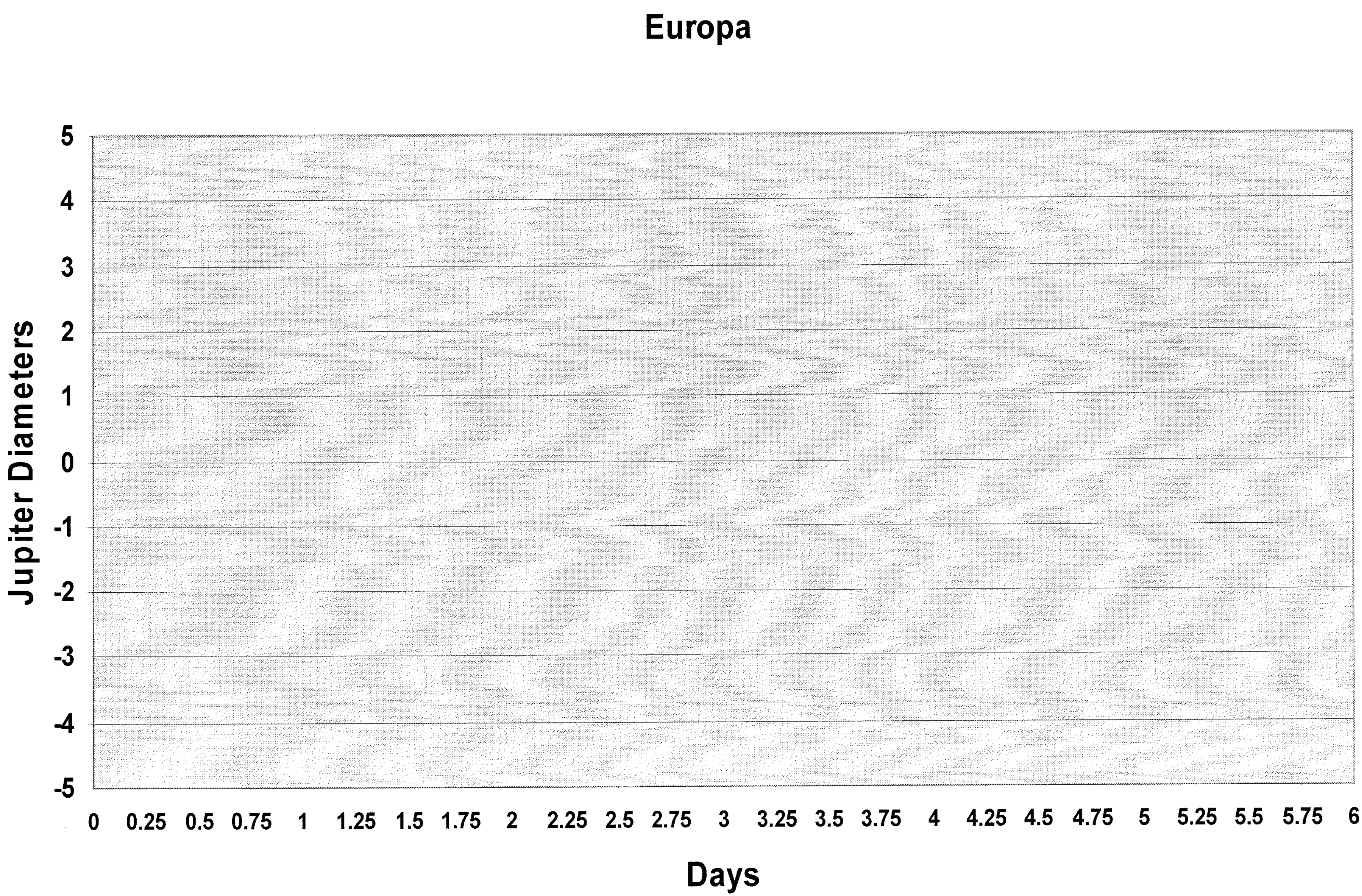
Data Table 1

	Ganymede	Europa
orbital period (days)		
orbital distance (JD's)		
orbital period (seconds)		
orbital distance (meters)		
mass of Jupiter (kg)		
average mass of Jupiter (kg)		

Data Table 2

mass of Earth (kg)	
mass of Sun (kg)	
$mass_{(Jupiter)}$ / $mass_{(Earth)}$	
$mass_{(Sun)}$ / $mass_{(Jupiter)}$	
$mass_{(Sun)}$ / $mass_{(Earth)}$	
$volume_{(Jupiter)}$	
$volume_{(Earth)}$	
$volume_{(Sun)}$	
$volume_{(Sun)}$ / $volume_{(Jupiter)}$	
$volume_{(Jupiter)}$ / $volume_{(Earth)}$	
$volume_{(Sun)}$ / $volume_{(Earth)}$	


Ganymede
Jupiter Diameters
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
Days


Europa
Jupiter Diameters
5
4
3
2
1
0
-1
-2
-3
-4
-5
0 0.25 0.5 0.75 1 1.25 1.5 1.75 2 2.25 2.5 2.75 3 3.25 3.5 3.75 4 4.25 4.5 4.75 5 5.25 5.5 5.75 6
Days