# Mathematics Review

### *Astronomy Laboratory Exercise*

# A

## Learning Objectives

In this exercise, students will:

- Review the necessary mathematics that is required to be successful in the astronomy laboratory class.

## Introduction

The astronomy laboratory class explores many different concepts in astronomy. The language of science is mathematics and must be used to express relationships and to calculate quantitative numbers in an attempt to understand the universe around us. The purpose of this exercise is to expose the student to the type of mathematics that will be used in this class and the level of difficulty that the mathematics will have. Most of the mathematics that is used will not go beyond the very basics of algebra and geometry.

The most useful tool you will use this semester is your scientific calculator. Use it wisely to help you quickly perform calculations that would otherwise be tedious (or impossible) to do long-hand. Much of the challenge for students that are being exposed to a scientific lab class for the first time is learning how to use their scientific calculator to perform the basic functions necessary for success in this class.

The following section is a basic introduction (or hopefully a reminder!) of the basic skills that will be used in the astronomy laboratory class.

## Exponents and your scientific calculator

An exponent is a shorthand way to express that a number has been multiplied by itself numerous times.

An example is $3^3 = 3 \times 3 \times 3 = 27$

Your scientific calculator has a button that will take care of this task for you. Each brand of calculator is slightly different, but most calculators will have either a $y^x$ or a $x^y$ button.

To perform $3^3$ on your calculator, you would press: | 3 | $y^x$ | 3 | = |

If the number has a negative exponent you would press one additional button.

An example is $3^{-3} = 0.037$

On your calculator, you would press: | 3 | $y^x$ | 3 | +/- | = |

## Scientific Notation

In the science of astronomy, as with most of the sciences, there are often very large numbers that are used in expressing values or in calculations. It is often inconvenient to express large numbers in a pure form. It is also dangerous to express very large numbers in their pure form for risk of omitting or adding an additional digit to the number and thus introducing an error to the calculation. To remedy this, scientists use a system called scientific notation to express numbers. This scientific "shorthand" is easy to use once you learn how the system works. Let's see how this is done.

***Example 1:*** The speed of light is 186,272 miles per second. In scientific notation it is expressed as $1.86\times10^5$ miles per second.

Scientific notation works by taking the whole number and expressing it as a number between 1 and 10 (but not including 10).

$$186{,}272 \longrightarrow 1.86$$ (rounding is typically done)

Next, figure out what power of ten the whole number is using. To do this, figure out how many decimal places is needed to turn 1.86 back into 186,272

$$1.86272 \rightarrow 10^5$$

putting it all together you get $1.86 \times 10^5$

If the number is less than one but larger than zero, there is a slightly different way to turn it into scientific notation shorthand.

***Example 2:*** The width of the finest human hair is about 0.0004 inches. In scientific notation it is expressed as $4\times10^{-4}$ inches.

Start by again taking the whole number and expressing it as a number between 1 and 10 (but not including 10).

$$0.0004 \longrightarrow 4$$

Again, figure out how many decimal places is needed to turn 4 back into 0.0004 $0.0004 \longrightarrow 10^{-4}$

but this time the number of decimal places is counted to the left and so the power of ten is a negative one.

putting it all together you get $4 \times 10^{-4}$

## Scientific Notation on your calculator

It is very important to start by understanding that you should **never use the $y^x$ or $x^y$ button** for scientific notation on your calculator! There is a button that is specifically designed for scientific notation.

For most calculators, it will look like **[EE]** or **[EXP]** Either button performs the $\times 10$ function.

If you wanted to enter $1.86\times10^5$ into your calculator, you would press the following buttons:

**[1] [.] [8] [6] [EE] [5]**

## Units and Unit Conversion

All measurements must have units. If somebody asked you how long you lived in your home town and you simply said "3", your answer would be very confusing. Do you mean 3 days? 3 months? 3 years? 3 decades? A number without a unit is meaningless.

In science, it is often necessary to change a measurement from one unit to another. Perhaps a distance is calculated in *kilometers* but you want to know it in *miles*. You might calculate the age of the Universe in *seconds* but it would be easier to understand if you knew it in *years*. There is a simple "cookbook" approach to converting from one unit to another. Let's see how this works.

***Here is an example***: The radius of the Earth is 6,378 kilometers. Suppose you want to know what the radius of the Earth is in miles?

**Step 1: Find a conversion factor and write it as a ratio:** ***1 mile = 1.61 km*** OR

$$\frac{1 \text{ mile}}{1.61 \text{ km}}$$

**Step 2: Find what units you want the final answer in:** ***miles***

**Step 3: Set up the equation so that the units you *don't* want will cancel out, and you are left with the units that you *do* want:**

$$6378 \cancel{\text{km}} \times \frac{1 \text{ mile}}{1.61 \cancel{\text{km}}}$$

and finally

$$6378 \times \frac{1 \text{ mile}}{1.61} = 3961 \text{ miles}$$

***Here is a second example:*** A telescope has a mirror that is 40 inches in diameter. Suppose you want to know what the diameter of the mirror is in *meters*?

**Step 1: Find a conversion factor, in this case there are *two*, and write them as ratios:**

$$1 \text{ m} = 100 \text{ cm} \qquad 1 \text{ inch} = 2.52 \text{ cm}$$

OR

$$\frac{1 \text{ m}}{100 \text{ cm}} \qquad \frac{1 \text{ inch}}{2.52 \text{ cm}}$$

**Step 2: Find what units you want the final answer in:** ***meters***

**Step 3: Set up the equation so that the units you *don't* want will cancel out, and you are left with the units that you *do* want:**

In this example you will need to flip the second conversion factor:

$$\frac{1 \text{ inch}}{2.52 \text{ cm}} \rightarrow \frac{2.52 \text{ cm}}{1 \text{ inch}}$$

putting it all together:

$$\frac{1 \text{ m}}{100 \cancel{\text{cm}}} \times \frac{2.52 \cancel{\text{cm}}}{1 \cancel{\text{inch}}} \times 40 \cancel{\text{inch}}$$

$$\frac{1 \text{ m}}{100} \times \frac{2.52}{1} \times 40 = 1.008 \text{ meters}$$

## Rearranging Equations

One of the most basic rules in algebra deals with rearranging equations. The rule is simple, *whatever function you perform on the one side of an equation, you must perform it on the other side too.*

***Here is a simple example:*** Suppose you start with the following equation $a = b\,c$

If your goal is to get the variable ***b*** by itself, then you must divide the right side by the variable that you *don't* want, in this case the variable ***c*** :

$$a = \frac{b\,c}{c}$$

but remember that you must also divide the left side by ***c*** as well: $\frac{a}{c} = \frac{b\,c}{c}$

so now the variable ***c*** will cancel out on the right side of the equation: $\frac{a}{c} = \frac{b\,\cancel{c}}{\cancel{c}}$

and you are left with: $b = \frac{a}{c}$

***Here is a second example:*** Suppose you start with the following equation: $x = \frac{y}{z}$

If your goal is to get the variable ***y*** by itself, then you must multiply the right side by the variable that you don't want, in this case the variable *z*:

$$x = \frac{y}{z}\,z$$

but remember that you must also multiply the left side by ***z*** as well: $z\,x = \frac{y}{z}\,z$

so now the variable ***z*** will cancel out on the right side of the equation: $z\,x = \frac{y}{\cancel{z}}\,\cancel{z}$

and you are left with: $y = x\,z$

**This rule/function is very useful and will be used quite extensively in the astronomy laboratory class!**

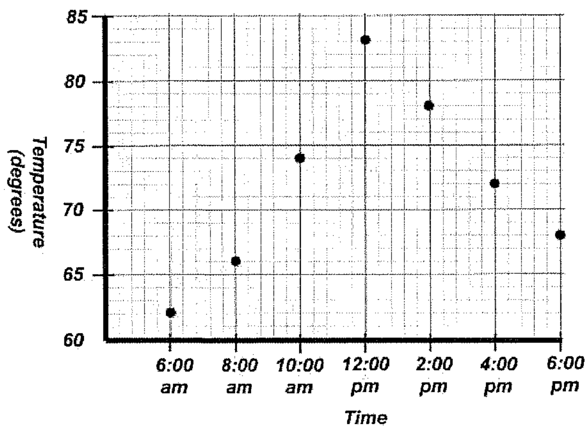
## The Power of Graphing

Graphing is a very powerful tool in science. Graphing is often used to display a visual representation of a concept, a function, or to help determine an unknown quantity. In the astronomy laboratory class we will use graphing *extensively* for a variety of purposes. One of the most important advantages to graphing is to be able to determine an approximate value to a function when the desired number is unknown. Let's see how this works.

***Here is an example:*** The outside temperature changes throughout the day. Table A (to the right) shows the readings of the outside air temperature at two-hour intervals on June 17th.

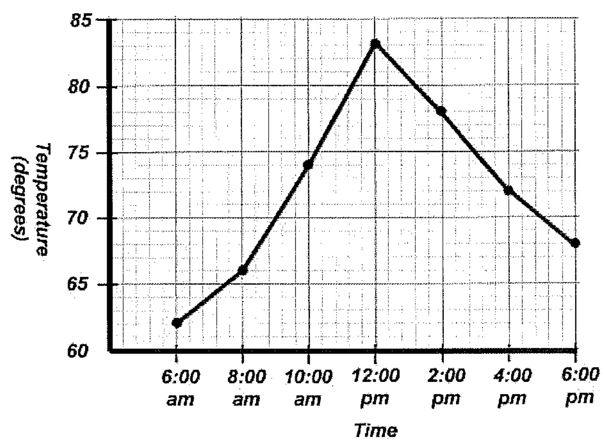
### Table A

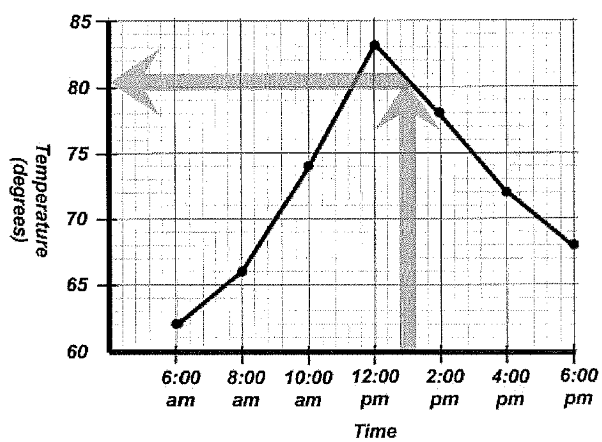
| Time of Day | Outside Air Temperature (degrees) |
|---|---|
| 6:00 am | 62 |
| 8:00 am | 66 |
| 10:00 am | 74 |
| 12:00 pm | 83 |
| 2:00 pm | 78 |
| 4:00 pm | 72 |
| 6:00 pm | 68 |




The graph to the left is a plot of the data from Table A. The time of day is plotted along the x-axis, and the outside air temperature is plotted along the y-axis.

A line is used to join all of the data points on the graph. The resulting line is now a _function_ of how the outside air temperature changed over time for June 17th.




A graph with a function can now be used as a tool. Suppose you wanted to know what the outside air temperature was for June 17th at 1:00 pm. Unfortunately, the temperature was not recorded at 1:00 pm on that day. However, using the line (which represents the *function* of air temperature vs. time) you can approximate what the outside air temperature was at 1:00 pm on that day: ***80.5 degrees***.

# Mathematics Review

**Name:**

***Astronomy Laboratory Data Sheet***

**<u>SHOW YOUR WORK! ATTACH ANY OTHER SHEETS IF NEEDED.</u>**

Express the following numbers in *scientific notation*. Use Appendix A for assistance if needed.

| | | | |
|---|---|---|---|
| 1. 5,280 | | 4. 93 million | |
| 2. 186,000 | | 5. 14 billion | |
| 3. 17,500 | | 6. 0.00418 | |

Perform the following calculations *using your scientific calculator.*

| | | | |
|---|---|---|---|
| 7. $(64.8)^3$ | | 9. $\sqrt{833}$ | |
| 8. $(11.86)^2$ | | 10. $\sqrt[3]{140.2}$ | |

Perform the following calculation *using your scientific calculator*. *Express your answers in scientific notation.*

| | | | |
|---|---|---|---|
| 11. $(1.73\times10^7)(3.2\times10^8)$ | | 13. $\frac{(9.8\times10^{12})(7.71\times10^4)}{3.14\times10^8}$ | |
| 12. $\frac{6.672\times10^{-11}}{5.56\times10^{23}}$ | | 14. $\frac{(9.4\times10^3)(5.67\times10^{10})}{(4.28\times10^5)(2.56\times10^3)}$ | |

Convert the values on the left into the units shown on the right. Use Appendix A for assistance.

| | | | |
|---|---|---|---|
| 15. $2.51\times10^{13}$ miles | light years | km | AUs |
| 16. 22 years | hours | minutes | seconds |
| 17. 45 degrees | arcminutes | arcseconds | |
| 18. 180 pounds | kilograms | | |

Rearrange the three equations to solve for the variables that are shown.

| | | |
|---|---|---|
| 19. d = vt | t = | v = |
| 20. $D = \frac{x}{A}$ | x = | A = |
| 21. $P^2 = d^3$ | P = | d = |

22. The Earth is 1 Astronomical Unit (AU) away from the Sun. The speed of light is 186,272 miles/sec. How many *minutes* does it take light to travel from the Sun to the Earth?

23a. Figure 1 (below) shows a map of the USA. The black line on the lower left gives the scale of the diagram. Measure the length of the line in millimeters. Determine how many miles there are in each millimeter.

The scale of the map is: ________miles/mm


Figure 1

23b. How many miles is it from San Diego to Denton Texas?

23c. How many miles is it from Denton to Cape Canaveral Florida?

23d. How many miles is it from San Diego to Cape Canaveral?

24a. The force of the Earth's gravity gets weaker the further away you get from the center of the Earth. Table A below contains data that represents the weight of a 180 lb person at various distances from the surface of the Earth. Using the graph paper below, graph the data contained in Table A.

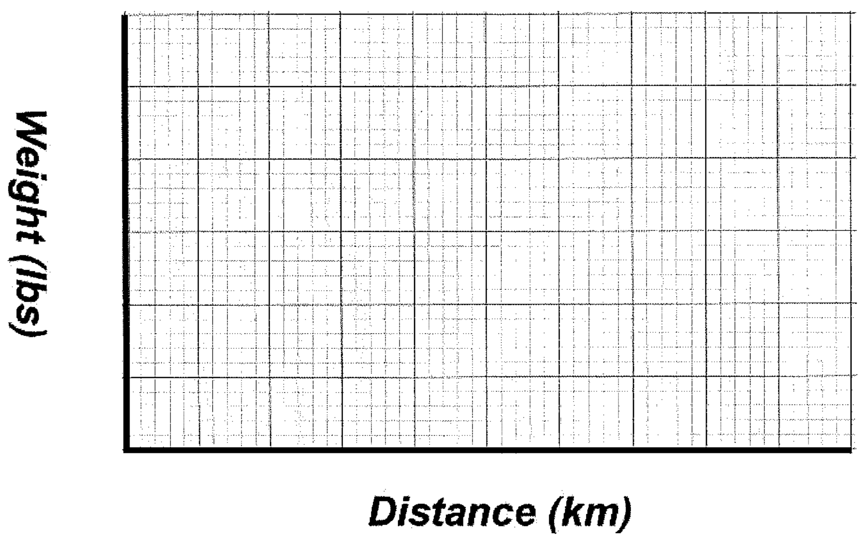



**Table A**

| Distance (km) | Weight (lbs) |
|---|---|
| 0 | 180 |
| 200 | 168 |
| 400 | 159 |
| 600 | 151 |
| 800 | 142 |
| 1000 | 135 |

24b. Draw a smooth line through all of the data points.

24c. What would the weight of the person be if they were at a distance of 500 kilometers above the surface of the Earth?

___________ lbs

24d. How high above the surface of the Earth would the person have to be to weigh 175 lbs?

___________ km