The Order and Distances of the Planets in the Solar System

Astronomy Laboratory Exercise

2

Learning Objectives

In this laboratory exercise, students will:

- Determine the sidereal periods of the planets in our Solar System.
- Determine the correct order of the planets in our Solar System.
- Determine the relative distances of the planets in our Solar System.

Definitions: planet, Astronomical Unit, inferior planets, superior planets, sidereal period, opposition, conjunction, elongation

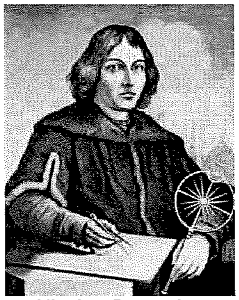
Review concepts of: constellations

Introduction

If you were to make an effort to view the night sky on a frequent basis, you would notice that the constellation patterns of the stars appear to be the same night after night. However you would also notice that a handful of stars appear to gradually change their positions against the backdrop of the constellations over a period of months or years. More than 2200 years ago, the ancient Greeks noticed that 5 "stars" in particular gradually moved along the patterns of constellations. The Greeks called these wandering stars πλανετοσ (or *planetos*) – the Greek word for "wanderers". Today we simply call them ***planets***. To the ancient Greeks, these five planets were nothing more than mysterious wandering stars. Over the centuries, we discovered that these five planets (now known to be a total of nine) are actually not stars at all, but instead are worlds unto themselves.

The ancient Greeks gave these "wandering stars" special names because they represented a unique part of the night sky. The names that have been given to the planets are related to ancient Greek and Roman mythology; many of the planets named after mythological gods and goddesses. Although the ancient Greeks did not realize that the planets were moving in orbits around the Sun (instead they believed that the planets – and the Sun – orbited around the Earth), they were still able to figure out the correct order of the planets according to their distances. How did they do that?

In the 15th Century, the mathematician/astronomer named Nicolas Copernicus was able to determine the *relative distances* of their planets in their orbits around the Sun. The *relative distance* is a comparison of the sizes of the orbits of the planets with the size of the Earth's orbital distance. By assuming that the Earth was a planet and that all of the planets orbited the Sun, Copernicus was able to use simple geometric relationships to determine the relative distances of the planets in their orbits. However in the 15th Century, nobody knew the <u>*true*</u> distance between the Earth and the Sun and so the actual distances of the planets could not be determined. Instead, Copernicus used the Earth – Sun distance (today called the ***Astronomical Unit***) as the unit of measure and related all of the orbital distances of the planets to that unit.



Nicolas Copernicus

In this exercise, you will explore and use the same logic that the ancient Greeks used to determine the correct order of the planets in our Solar System. Then you will explore and use the geometry that Copernicus used to determine the relative distances between the planets and the Sun. In a future exercise, you will explore the method that astronomers use to determine the *true* size of the Astronomical Unit and convert the relative orbital distances to true distances in miles.

The Order of the Planets

The ancient Greeks were able to determine the order of the planets by observing how they moved through the constellation patterns. They observed that not all the planets moved at the same speed, night after night, through the sky. Some planets made obvious changes against the backdrop of constellations each night, whereas others took months or years to make noticeable progress. As a result, the ancient Greeks reasoned that the faster moving planets were closer (and hence moving more noticeably) and the slower moving planets were further away. This allowed them to order the planets by ranking them according to their observed speeds. It worked! In addition, using simple logic, the ancient Greeks were able to deduce that a planet would complete one orbit when it would return to its original starting point in the sky. From this, they were able to determine the orbital periods of the planets in our Solar System. Let's explore how this was done.

The planets in our Solar System are divided into two groups. The planets closer to the Sun than the Earth are called ***inferior planets***, whereas the planets that are further than the Earth are referred to as ***superior planets***.

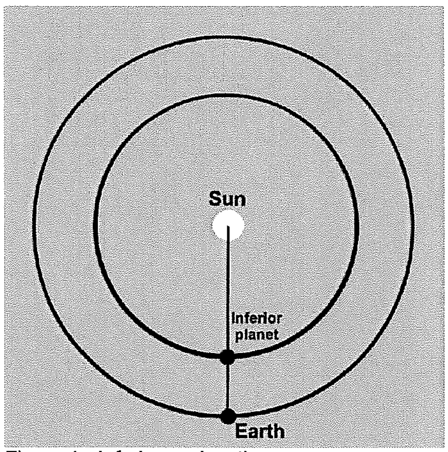


Figure 1: Inferior conjunction

There are several key alignments of the planets that will routinely occur. When an inferior planet lines up between the Earth and the Sun, it is called ***inferior conjunction*** (see Figure 1). If you were able to see the planet in the sky (see Figure 2), you would see it in the same direction as the Sun (or if the conjunction alignment was perfect, you would see the planet cross in front of the Sun).

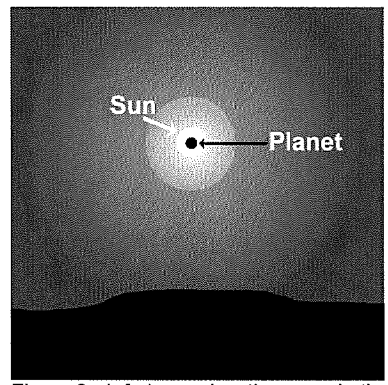


Figure 2: Inferior conjunction seen in the sky

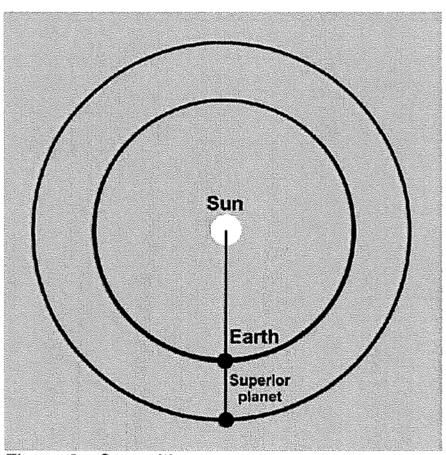


Figure 3: Opposition

When a superior planet lines up in its orbit opposite the Earth and the Sun (see Figure 3), an alignment called ***opposition*** occurs. This can sometimes be the closest distance that the Earth and a superior planet will get to each other in their orbits. The view of a planet at opposition in the sky is more subtle to the naked eye. At opposition, a superior planet will be seen to cross the meridian (the half way point of the sky) at midnight (see Figure 4). Think for a moment, can you figure out why this crossing of the meridian occurs at *midnight*?

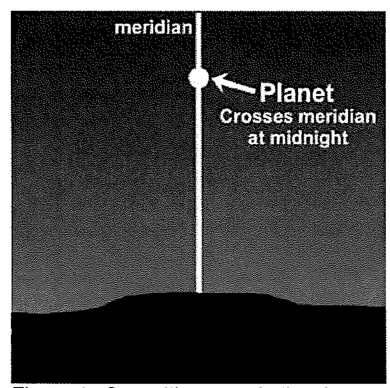


Figure 4: Opposition seen in the sky.

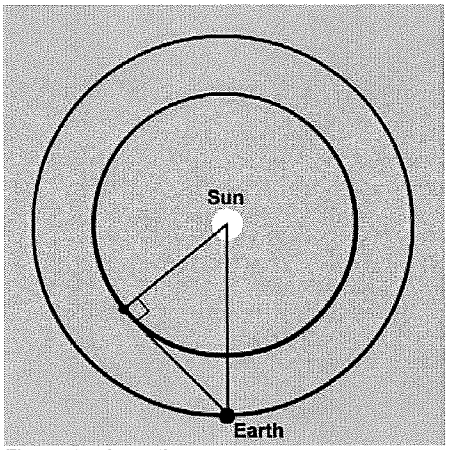


Figure 5: elongation

One final important planetary alignment occurs when an inferior planet is seen to be the furthest point, east or west, of the Sun. This alignment is called ***elongation*** (see Figure 5). If the inferior planet is east of the Sun, it is called *greatest eastern elongation*; if the planet is west of the Sun, the alignment is called *greatest western elongation.* Seen in the sky, elongation occurs when an inferior planet is seen at a point where it is the furthest distance away from the Sun (either east or west of the Sun).

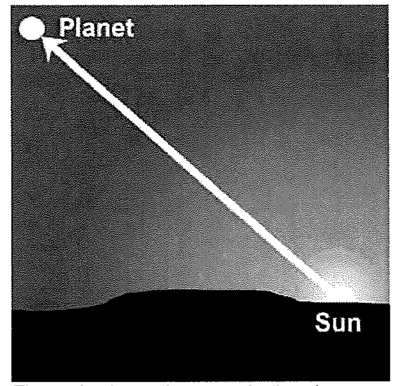


Figure 6: elongation seen in the sky.

Copernicus recognized that only inferior planets will demonstrate elongations and inferior conjunctions with respect to the Earth, and only superior planets will demonstrate oppositions.

Procedure

The ***sidereal period*** of a planet refers to how long it takes for a planet to go from lining up with a distant star, completing one full journey around the sky, and re-aligning with the original distant star. This also represents how long it takes a planet to complete one orbit around the Sun. The ***synodic period*** of a planet refers to how long it takes a planet to line up with the Sun as seen by observers on Earth (either at inferior conjunction or opposition) on two successive occasions. In Figure 7, Mercury and the Earth begin by both aligning to a distant star and the Sun (Figure 7 – left frame). Next (in the middle frame), Mercury completes one full orbit of the Sun - its sidereal period - and has realigned with the same distant star, however the Earth has also moved a bit in its orbit around the Sun and so Mercury must complete more than one orbit to catch up to the Earth a second time. Finally (in the right frame) it takes Mercury 115.9 days to realign with the Earth and Sun a second time (completing one synodic period).

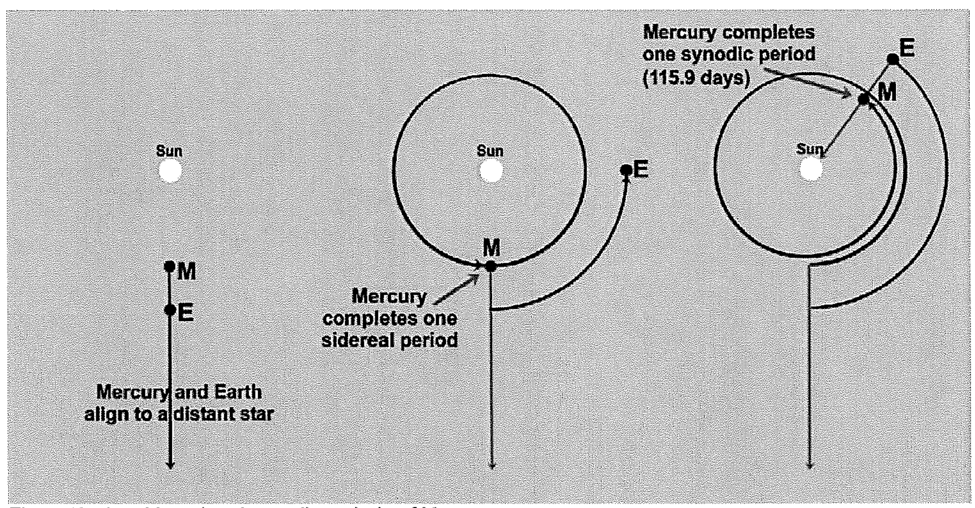


Figure 7: the sidereal and synodic periods of Mercury

Copernicus discovered that there is a simple mathematical formula that relates the sidereal period of a planet, the sidereal period of the Earth, and the synodic period of a planet (see Equation 1 below). From this equation, the sidereal period of a planet can be determined if an observation of how long it takes for the Earth and the planet to line up with the Sun two successive times (the synodic period of the planet) is made.

$$\frac{1}{P}=\frac{1}{S}+\frac{1}{P_E}$$

Equation 1
(Inferior planets)

where P is the sidereal period of a planet (in days), P_E is the sidereal period of the Earth (in days), and S is the synodic period of the planet (in days). The equation is slightly different for planets that are *superior* planets. Equation 2 shows the relationship for superior planets:

$$\frac{1}{P}=\frac{1}{P_E}-\frac{1}{S}$$

Equation 2
(Superior planets)

Data Table 1 (on the Data Sheet) lists all nine planets in *alphabetical order*. The table also lists whether the planet is inferior or superior and the synodic period for each planet.

1. The sidereal period (P_E) of the Earth is 365.25 days. This is how long it takes the Earth to complete one orbit around the Sun. Using the synodic periods in Data Table 1 and either Equation 1 or Equation 2, calculate (in days) the sidereal periods, (<u>*P*</u> in the equation), of all eight planets other than the Earth. Be sure to use the appropriate equation based upon if the planet is inferior or superior. NOTE: Carry your numbers to at least 6 decimal places to ensure you get an accurate answer. *Record your answers in Data Table 1 on the Data Sheet.*

2. The ancient Greeks were able to rank the planets in order, nearest to farthest, by how long they took to make one complete journey across the sky (as seen against the background stars). Recall that this is the sidereal period of a planet. Using the sidereal periods of all nine planets (that you calculated in step 1) rearrange the nine planets into their correct order going from *shortest* sidereal period to *longest* sidereal period, and list them (in order) *in Data Table 2 on the Data Sheet.*

3. Convert each of the sidereal periods (originally calculated in days in step 1) to years, *listing your answers in Data Table 2 on the Data Sheet.*

<u>The Distances to the Planets</u>

To determine the relative distances to the planets, Copernicus used simple geometric relationships and basic observations of the planets. However, these relationships were only able to tell astronomers how far each planet is from the Sun *relative* to the Earth's orbit around the Sun (Recall that the distance between the Earth and the Sun is referred to as the Astronomical Unit). In the 15^{th} Century, the <u>*true*</u> distance between the Earth and the Sun was not known (and would not be known for hundreds of years) and so the true sizes of the orbits of the planets in our Solar System would not be known either. Although knowing the true distances between the planets was preferred, astronomers still desired to know how the sizes of each planet's orbit compared to the others (and the Earth's) and so this did not deter Copernicus and others from determining this information. Let's take a look at how the relative distances between the planets can be found.

4. Figure 8 (below), shows the planets Mercury and Venus at greatest eastern elongation in their orbits. Measure the distance between the center of the Sun and the center of each planet. Using the scale on the bottom of each figure (Note: the scales are different for each image!), determine the angles between the Sun & Mercury, and the Sun & Venus when they are at greatest eastern elongation. *Record your answers in Data Table 1 on the Data Sheet.*

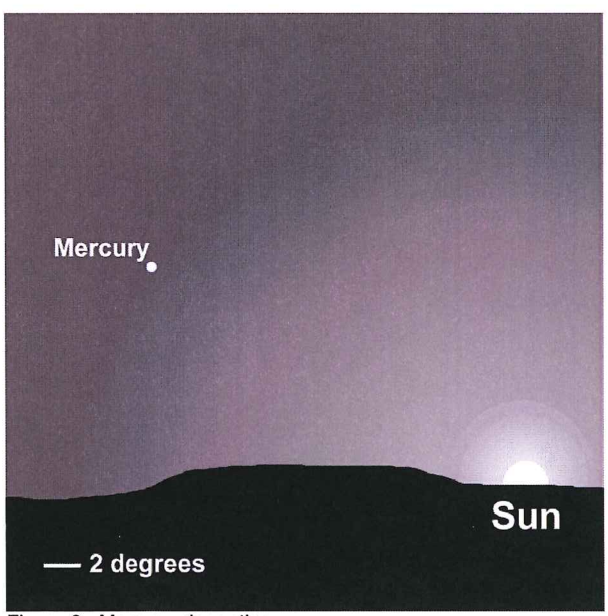
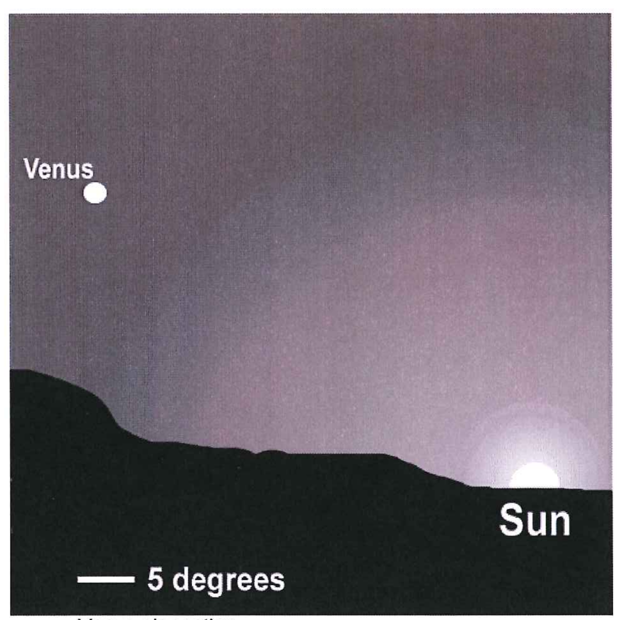


Figure 8: Mercury elongation



Venus elongation

5. Figure 9 shows the geometrical relationship that relates the angle between a planet and the Sun (when it is at elongation), the distance between the planet and the Sun, and the distance between the Earth and Sun in Astronomical Units. It is used to determine the relative distance between a planet and the Sun. Equation 3 is the resulting formula that is used to calculate the relative distance from the planet to the Sun. Calculate the relative distances between Mercury and the Sun, and Venus and the Sun using Equation 3 and your measurements from step 4. *Record your answers in Data Table 2 on the Data Sheet.*

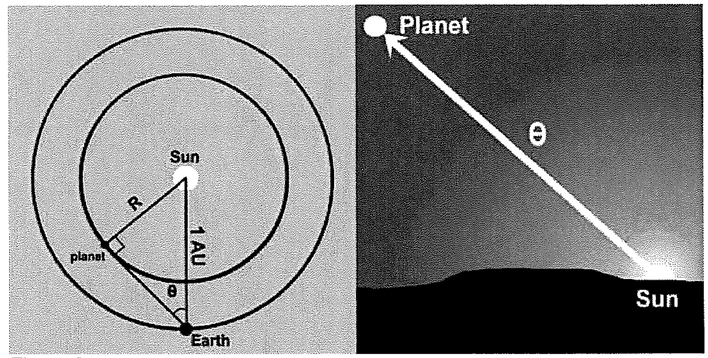


Figure 9

$$R = \sin(\theta)$$

Equation 3

where θ is the elongation angle between the planet and the Sun (see Figure 9, right).

To determine the relative distance to a *superior* planet, Copernicus determined that two observations of a planet were necessary. Figure 10 shows the Earth and Mars at opposition. One full Martian orbit later, Mars is back to where it started, but the Earth has already completed one orbit of the Sun and is most of the way around a second time (see Figure 10, middle). If the angle between the Sun, Earth and Mars (referred to as the *elongation angle* of Mars) is measured, and the angle between the Earth, Mars and the Sun is measured, then the "law of sines" can be used and the distance between Mars and the Sun can be determined.

6. On February 2nd, 1999 Mars was 100 degrees away from the Sun as seen in Earth's sky (this was its elongation angle - see Figure 10, middle). The angle between the Earth, Mars and the Sun was 41 degrees (see Figure 10, middle). Using Equation 4 determine the relative distance between Mars and the Sun. *Record your answer in Data Table 2 on the Data Sheet.*

$$Mars\ \ to\ \ Sun\ \ dist. = \frac{\sin(elongation\ \ \ angle)}{\sin(41^\circ)}\ x\ \ 1AU$$

Equation 4

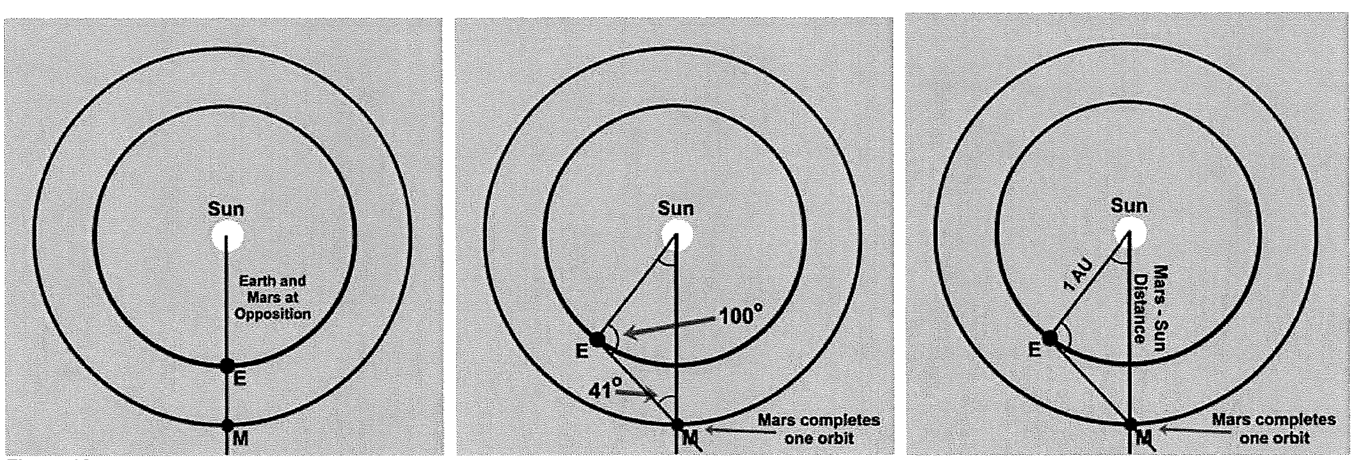
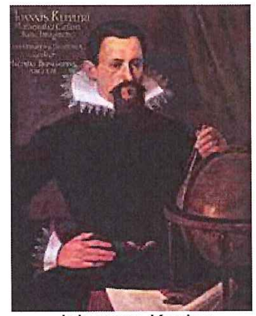


Figure 10

An Important Discovery about Planetary Distances

In the early part of the 17th Century, a mathematician/astronomer named Johannes Kepler, discovered a relationship between the orbital period of a planet and its orbital distance (from the Sun). This relationship has become known as *Kepler's Third Law* (the details of this law and his other discoveries will be discussed in a future exercise). In this relationship, the sidereal period of a planet (measured in Earth years) is related to its orbital distance measured in Astronomical Units. Equation 5 (below) shows Kepler's Third Law:



Johannes Kepler

$$P^2 = d^3$$

Equation 5

where <u>P</u> is the sidereal period of a planet (in Earth years), and <u>d</u> is the orbital distance of a planet (in AUs)

7. Using Equation 5 and the sidereal periods of Jupiter, Saturn, Uranus, Neptune and Pluto from Data Table 1, calculate the orbital distances of these planets in Astronomical Units and *record their values in Data Table 2 on the Data Sheet.*

Distance to the Moon

The Moon is the Earth's only natural satellite. Long ago, the ancient Greeks determined that the Moon indeed orbits around the Earth, but they did not know how far away the Moon was. The distance between the Earth and Moon can be determined using a bit of geometry. If the Moon is observed simultaneously from two different places on Earth, then geometrical triangulation can be used to calculate its distance.

Figure 11 (below) shows the geometry involved in determining the Moon's distance. When seen from two different locations on Earth, the moon is seen at two slightly different angles when measured against the background of stars. The geometry of the situation yields the distance between the Earth and the Moon as shown in Equation 6.

$$D = \frac{x}{A} 206265$$

Equation 6

where <u>A</u> is measured in ***arcseconds***.

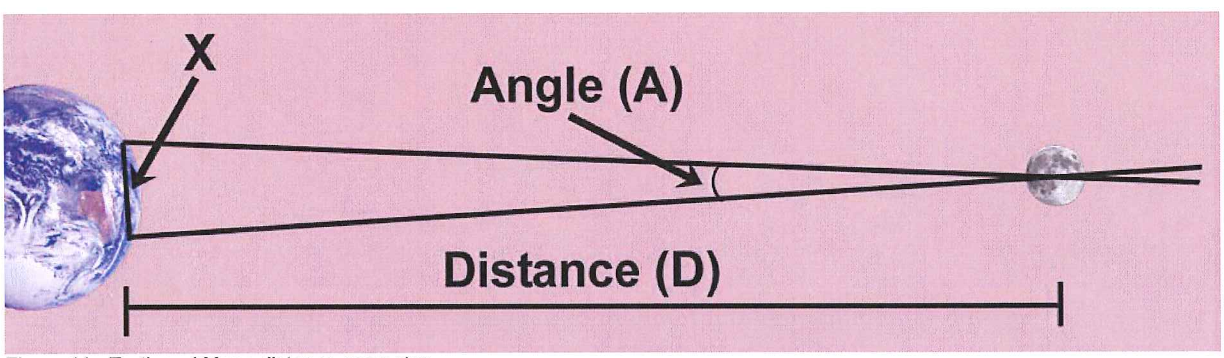


Figure 11: Earth and Moon distance geometry

8. The Moon is seen simultaneously from Honolulu, Hawaii and New York City (6,000 miles away). The angular difference in position of the Moon seen against the background of stars (angle <u>A</u> in Figure 11) is 5290 arcseconds (1.47 degrees). Use Equation 6 to determine the distance between the Earth and the Moon. *Record your answer in Data Table 3 on your Data Sheet.*

9. Finally, using the polar graph paper included with this exercise, plot the relative distances between each planet and the Sun. Use the orbital distances that you calculated in AUs (listed in Data Table 2). The location of the Sun is at the center of the polar graph paper (where all the lines converge). Draw half-circles at the distances of each planet to show their orbits around the Sun, and label each orbit with the planet's name.

The Order and Distances of the Planets in the Solar System

Name:

Astronomy Laboratory Data Sheet

Data Table 1

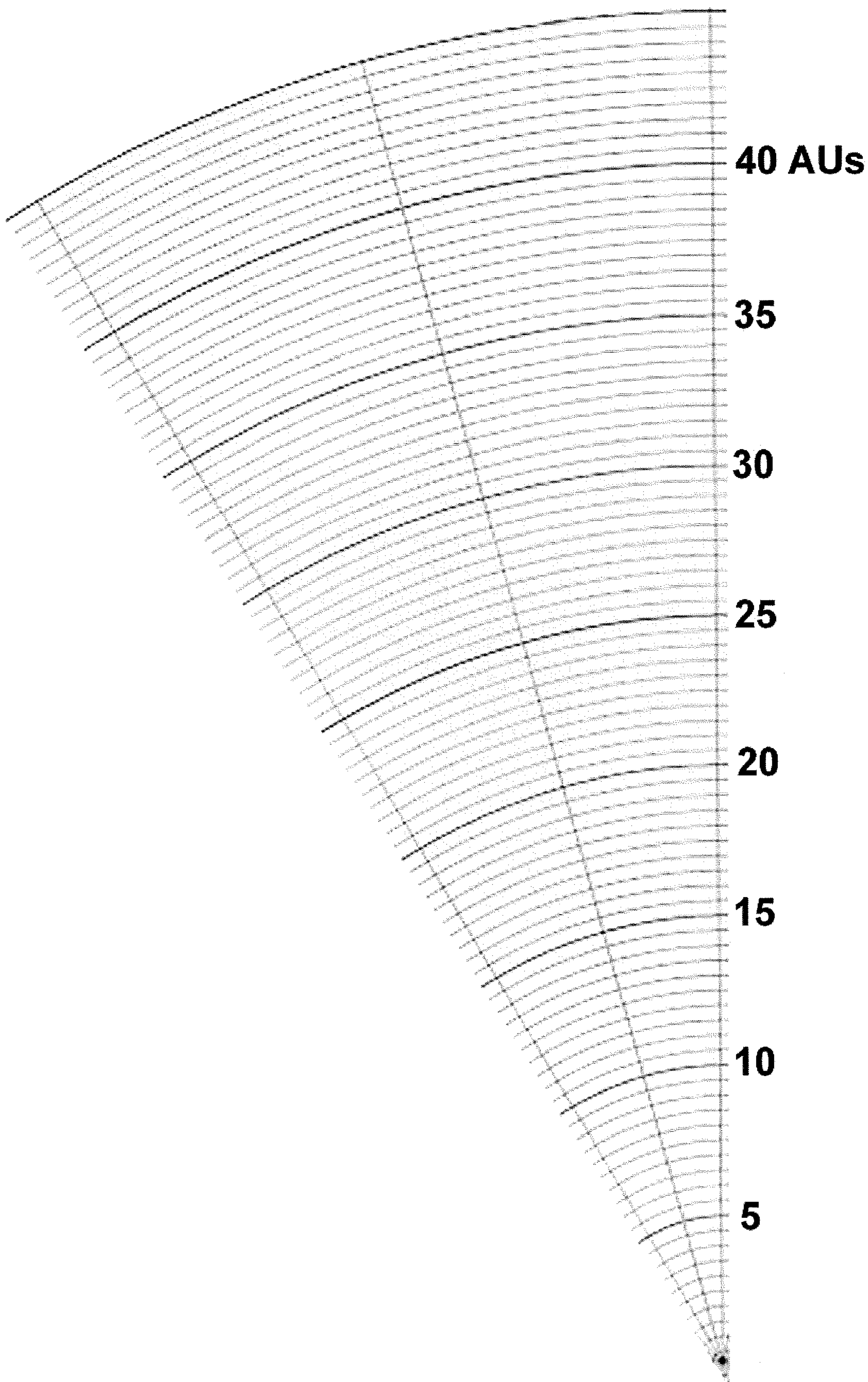
Planet Name	Inferior (I) Or Superior (S)	Synodic Period	Sidereal Period (days)	θ
Earth	---	---	365.25 d	
Jupiter	S	398.9 days		
Mars	S	779.9 days		
Mercury	I	115.9 days		
Neptune	S	367.5 days		
Pluto	S	366.7 days		
Saturn	S	378.1 days		
Uranus	S	369.7 days		
Venus	I	583.9 days		

Data Table 2

Planet Name	Sidereal period (years)	Distance from the Sun (AUs)

Data Table 3

Distance to the Moon (miles)


40 AUs
35
30
25
20
15
10
5