The Transit of Venus: The True Size of the Astronomical Unit

Astronomy Laboratory Exercise

3

Learning Objectives

In this laboratory exercise, students will accomplish:

- Determine the true size of the Sun.
- Determine the true size of the Astronomical Unit.
- Determine the true sizes of the orbital distances to each planet in our Solar System.

Definitions: transit

Review concepts of: Astronomical Unit, inferior planet, inferior conjunction

Introduction

Using basic geometry and astronomical observations, Nicholas Copernicus was able to determine the relative sizes of the orbits of the planets (see *"The Order and Distances of the Planets in the Solar System"* exercise). However, It is impossible to determine the true sizes of the distances each planet is from the Sun without knowing the size of the Astronomical Unit. Until the 20th century, without the aid of modern technology, determining the size of the Astronomical Unit was only possible by observing a transit event of either the planets Mercury or Venus.

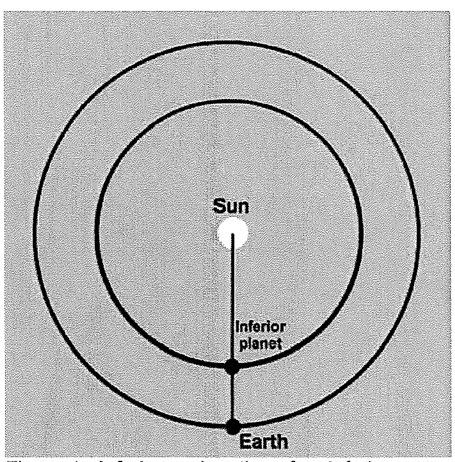


Figure 1: Inferior conjunction of an inferior planet.

A planetary ***transit*** occurs when one solar system body passes in front of another. This could be a moon of a planet passing in front of the planet (as seen by observers on Earth) or it could be a planet seen passing in front of another planet. However a particular type of transit occurs at *inferior conjunction* (see Figure 1) of a planet, when the planet is seen to pass in front of the Sun (see Figure 2). This is only possible for the planets Mercury and Venus since their orbits lie within the orbit of the Earth around the Sun. If the geometry of the orbits of the Earth and either Mercury or Venus are just right, a transit occurs and can be witnessed by earthly observers.

Because Venus has an orbit around the Sun that is tilted with respect to the orbit of the Earth, alignments that produce a transit are exceedingly rare. The transit of Venus across the face of the Sun occurs with a strange repeating pattern. A transit will occur in early December of a particular year, then eight years later another transit will occur (again in early December). A period of 121 years will then elapse before the next transit (occurring in early June) with another eight year wait until the next transit (again occurring in early June). Finally 105 years will go by before the whole pattern repeats again[1].

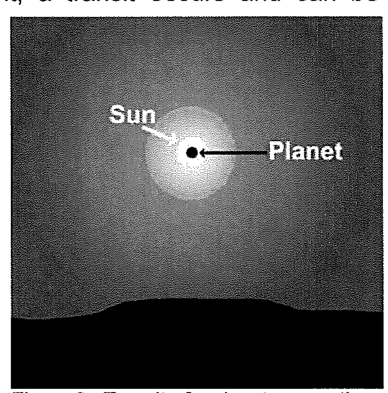


Figure 2: Transit of a planet across the face of the Sun as seen in the sky.

[1] Fred Espenak, NASA/GSFC

The first predicted transit of Venus occurred in 1631 when Johannes Kepler accurately predicted a transit of Venus in December of that year. However, the transit occurred after sunset for Europe, and so nobody was able to observe the event. Eight years later, an astronomer named Jeremiah Horrocks (using the same method that Kepler employed) predicted a transit for December 4th 1639. Horrocks, using a simple telescope that projected the image of the Sun safely onto a sheet of paper, became the first person to observe such a rare event.

In the year 1663, the mathematician, James Gregory, first suggested that a transit of Venus (or Mercury) could be used to accurately determine the size of the Astronomical Unit. However it wouldn't be until the next Venus transit of 1761 that the observations and measurements could be made. In order to be able to calculate the true distance of the Astronomical Unit, two measurements must be made of the same transit from two different latitudes on Earth. Unfortunately, the inability of measuring the precise location of the observers involved in the 1761 transit lead to an inaccurate measurement of the Astronomical Unit. Future transit observations using better technology (in particular observations made after the invention of photography) would eventually yield the precision necessary to refine the accuracy of the true distance of the Astronomical Unit.

In the first part of the 21st Century, there will be two opportunities to witness a transit of Venus. The first was on June 8th, 2004 and the second will be on June 6th, 2012. In this laboratory exercise, you will use the data gathered from the June 8th, 2004 transit to graph the position of Venus as it was seen crossing the face of the Sun as witnessed from two different locations on Earth. From this diagram, you will be able to make measurements that will lead you to a calculation of the true size of the Sun, and the true size of the Astronomical Unit.

Procedure

The method that is used to determine the size of the Astronomical Unit requires knowing the relative distance from Venus to the Sun (in Astronomical Units) and observations of the transit of Venus seen from two widely separated latitudes on Earth.

1. Examine the transit geometry diagram (Figure 4 on page 35) and the description of the transit geometry of Venus.

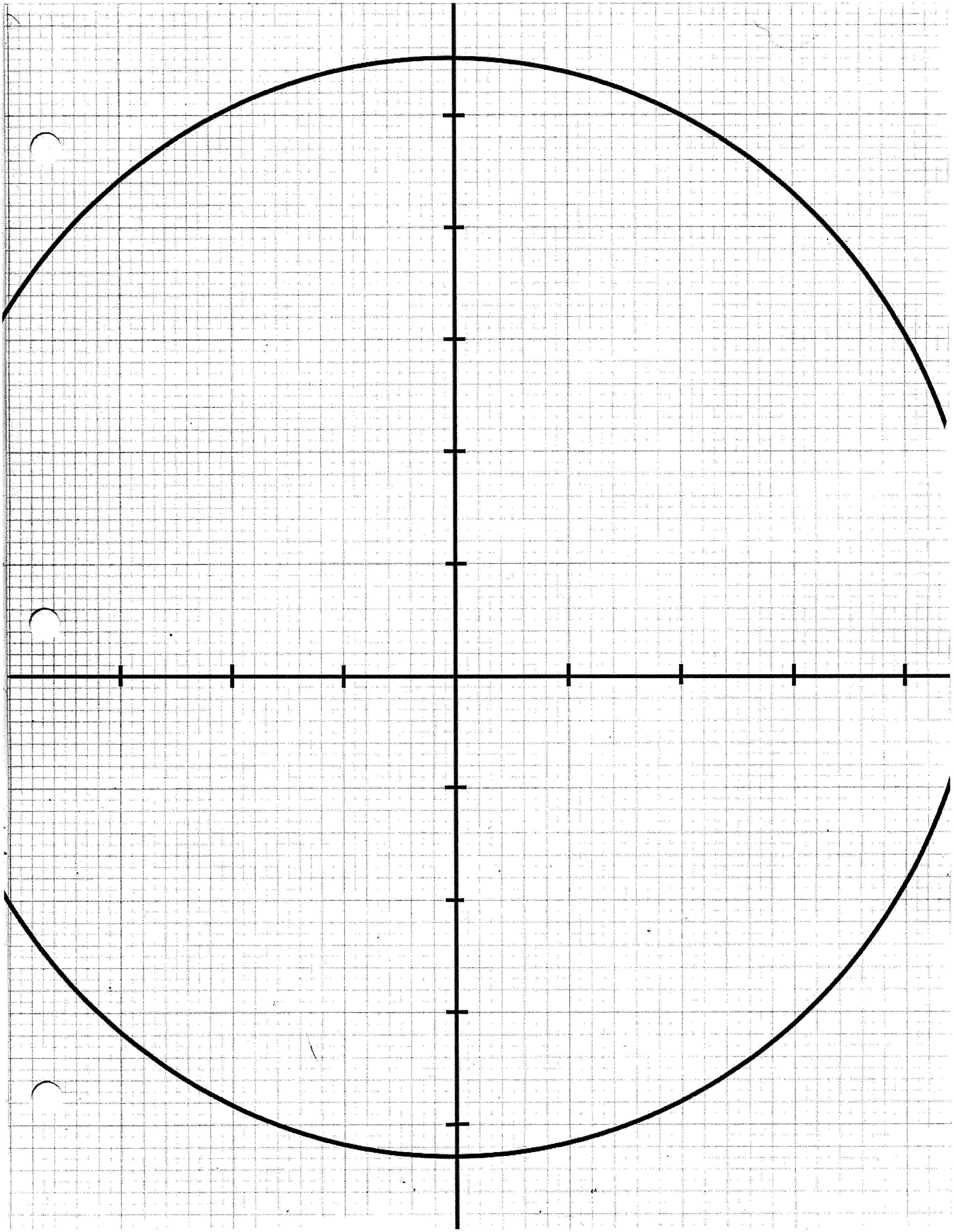
2a. The two locations that were used to observe this transit were Miami, Florida, and Santiago, Chile. Using the graph provided on page 32, plot the observations of the June 8th, 2004 transit of Venus listed in Table A below. (Note: you do _not_ need to label your data points with the time of the observation.) Be sure to plot the data for each location separately.

2b. Carefully connect the data points with your ruler. ***Draw your line as narrowly and as carefully as possible making sure to accurately pass through each data point.***

Table A

Date	Time	Location A (Miami)	Time	Location B (Santiago)
June 8, 2004	7:20	(-24, -18)	7:06	(-28, -16)
June 8, 2004	7:39	(-18, -19)	7:24	(-23, -17)
June 8, 2004	8:01	(-8, -21)	8:09	(-7, -20)
June 8, 2004	9:43	(+8, -24)	8:47	(-2, -21)
June 8, 2004	10:12	(+13, -25)	9:22	(+4, -22)
June 8, 2004	10:48	(+23, -27)	10:16	(+14, -24)

Table A: The x, y coordinates of the location of Venus across the disk of the Sun for the June 8th, 2004 transit.



3. In a previous lab, you calculated that the distance from Venus to the Sun (in Astronomical Units) is 0.72 AU's. From this, at the time of the transit, the distance from the Earth to Venus is 0.28 AU's. The key in being able to calculate the distances involved is related to the concept of similar triangles in geometry (see below):

Figure 3 (to the right) shows three nested right triangles (adZ, beZ, cfZ). Simple geometry shows that if all three triangles are right triangles, then the following relationship is true:

$$\frac{da}{dZ} = \frac{eb}{eZ} = \frac{fc}{fZ}$$

Equation 1

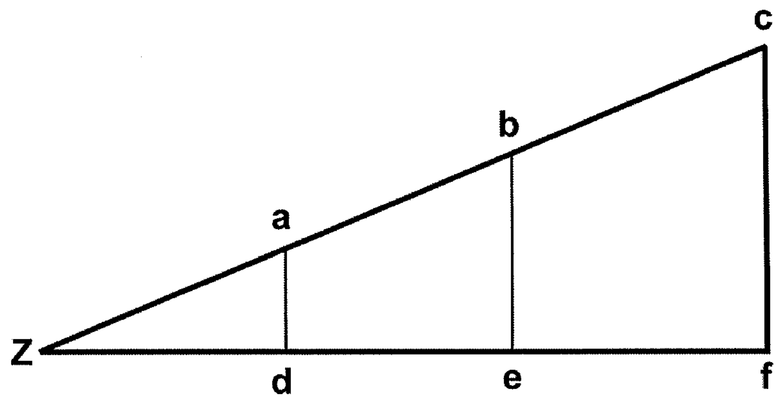


Figure 3: similar triangles.

Using the tangent law for right triangles in geometry, for any angle (except the 90° angle) the side opposite the angle divided by the side adjacent the angle always yields the same number.

With this concept in mind, consult Figure 4 on page 35. In Figure 4, there are two similar triangles. The triangle that starts at Venus and has a base at the Sun (S_T) is similar to the triangle that also starts at Venus and has its base on the Earth (S_o). The distance from Venus to the Sun (0.72 AU's) is the altitude of the first triangle. The distance from Earth to Venus (0.28 AU's) is the altitude of the other, similar, triangle. From geometry, the base divided by the altitude of one triangle is equal to the base divided by the altitude of a similar triangle. From this, the relationship in Equation 1 can be used to give:

$$\frac{S_o}{0.28\ AU} = \frac{S_T}{0.72\ AU}$$

Equation 2

4. The data provided in Table 1 is from Observer A located in the city of Miami. The data provided in column 2 of Table 1 is from Observer B located in the city of Santiago. Both cities have roughly the same longitude but are separated in latitude by a distance of 6480 kilometers. This distance is the value for (S_o). Using Equation 2, determine the size of S_T. *Record your answer in Data Table 1 on the Data Sheet.*

Use the example shown in Figure 5 to the right for guidance in steps 5 – 7 *(make your measurements on your graph).*

5. Measure the distance (in millimeters) on your graph paper between the two transit paths that were observed by Observer A and Observer B. *Record your answer in Data Table 1 on the Data Sheet.*

6. Measure the diameter of the disk of the Sun on your graph (in millimeters). *Record your answer in Data Table 1 on the Data Sheet.*

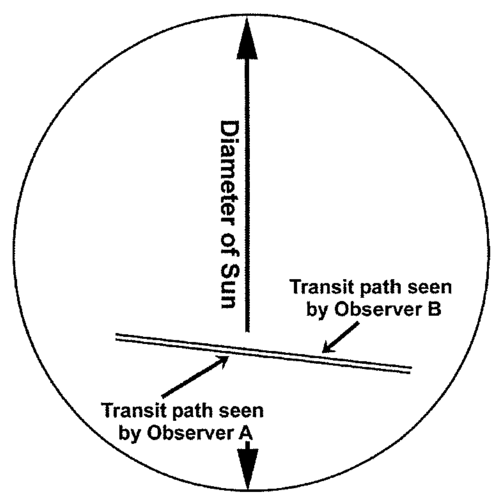


Figure 5

7. The distance between the two transit paths across the disk of the Sun compared with the diameter of the disk of the Sun (on your graph) should be the same as the transit separation (S_T) compared with the true diameter of the Sun. Using the following relationship determine the true diameter of the Sun ***in kilometers*** (note: you must rearrange Equation 3 to solve for the *"Sun's true diameter")*:

$$\frac{Circle\ diameter}{Drawing\ Separation} = \frac{Sun's\ true\ diameter}{Transit\ Separation\ (S_T)}$$

Equation 3

This is how astronomers first determined the true size of the Sun! *Record your answer in Data Table 1 on the Data Sheet.*

8. Divide the Sun's true diameter by 2 to get the radius of the Sun. *Record your answer in Data Table 1 on the Data Sheet.*

9. One last geometrical relationship can now be used to determine the true size of the Astronomical Unit. Figure 6 (below) shows a triangle that uses the tangent relationship (see step 3 above) again:

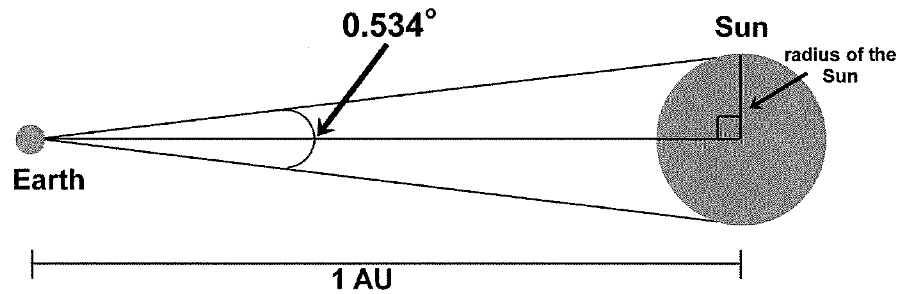


Figure 6

10. The apparent angular diameter of the Sun (as seen in the sky) can be very accurately measured. The average angle is 0.534°. The triangle (shown above in Figure 6) is a right triangle. The narrow angle is half of 0.534°, and the side that is opposite that angle is the radius (half the diameter) of the Sun. The Earth-Sun distance (Astronomical Unit) is the side adjacent to the small angle and can be calculated using the tangent definition. Determine what half of 0.534° is and *record your answer in Data Table 1 on the Data Sheet*

11. Finally, use Equation 4 below to determine the true size of the Astronomical Unit (note: you will have to rearrange the equation to solve for 1 AU) *Record your answer in Data Table 1 on the Data Sheet.*

$$\tan\left(\frac{0.534^\circ}{2}\right) = \frac{radius\ of\ Sun}{1\ AU}$$

Equation 4

12. Convert your answer in step 10 from kilometers to miles.

13. Refer to the results of the *"The Order and Distances of the Planets in the Solar System"* exercise. Transfer names of planets (in the correct order) and the relative distances you determined in that exercise to Data Table 2 on the Data Sheet of this exercise.

14. Using Data Table 2 on the Data Sheet of this exercise, convert the relative distances to the planets (previously determined in AU's) to <u>kilometers</u> using your newly determined value for the true size of the Astronomical Unit. *Record your answers in Data Table 2 on the Data Sheet.*

Transit Geometry

Observer A, at a particular known latitude, observes the transit of Venus and records their observations. **Observer B**, at *a very different latitude*, witnesses the same event and records their observations. Each observer then plots the path that Venus takes across the disk of the Sun. The two different observed paths Venus takes across the face of the Sun is caused by the observers witnessing the transit from two different vantage points on Earth. If the gap between the two different paths of the transit is measured (**S_T**), and the distance between the observer's location on Earth is known (**S_o**), then the relative distance between Venus and the Sun can be converted into a real distance. From this the size of the Astronomical Unit can be calculated.

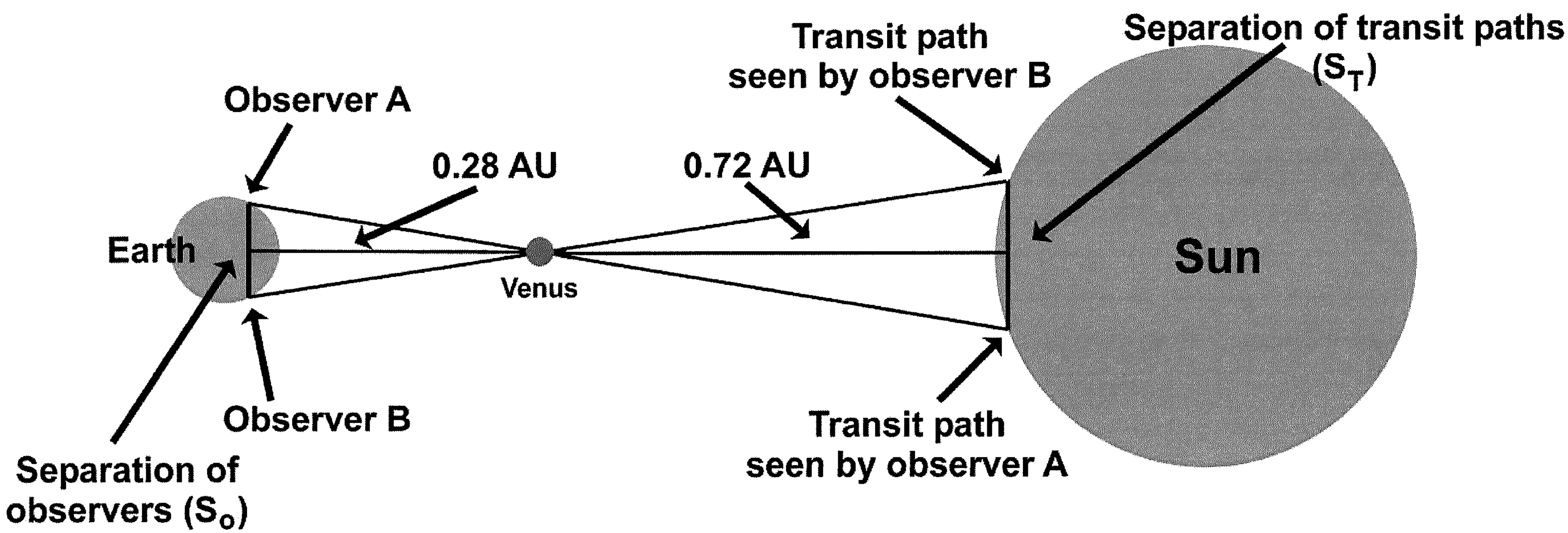


Figure 4: The transit geometry of Venus.

The Transit of Venus: The True Size of the Astronomical Unit

Name:

Astronomy Laboratory Data Sheet

Data Table 1

S_T	
Distance between the two transit paths	
Size of the disk of the Sun (from your graph)	
Sun's true diameter	
Sun's radius	
Half of the Sun's angular diameter	
True size of the Astronomical Unit (km)	
Astronomical Unit (miles)	

Don't forget to include units in your answers!

Data Table 2

Planet	Relative distance (AU's)	True distance (kilometers)